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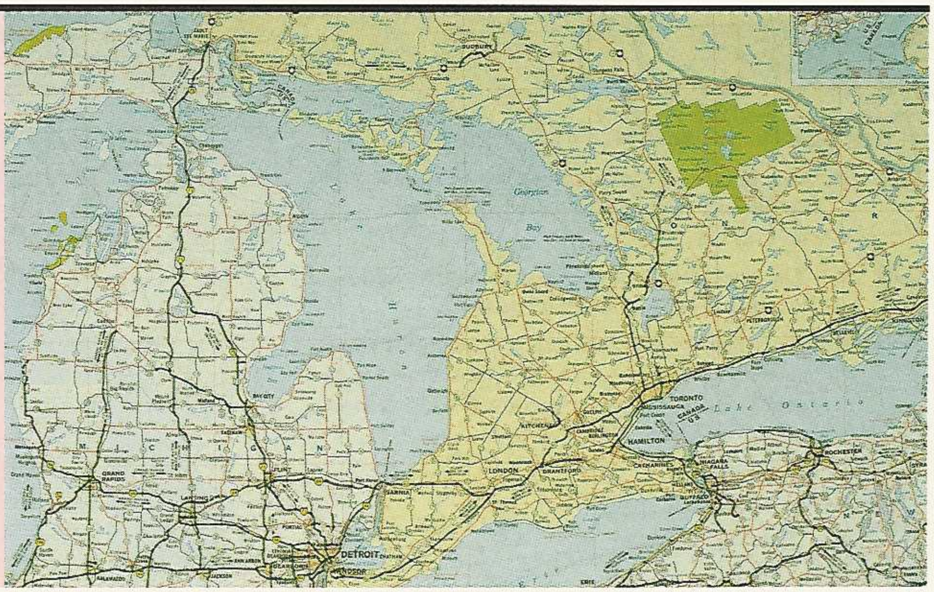
CHAPTER

5 Rational Expressions, Ratio and Proportion

On a road map, 3 inches represents a distance of 45 miles. If the distance between Detroit and Sault Ste. Marie is 23 inches on the map, how far is it from Detroit to Sault Ste. Marie?



5–1 ■ Rational numbers and rational expressions

A rational expression

In chapter 1, we defined a rational number.

Rational numbers

A rational number is a number that can be written as the quotient of two integers with the divisor (denominator) not zero.

Examples of rational numbers are

$$6, \quad \frac{3}{4}, \quad \frac{-5}{6}, \quad \text{and } \frac{8}{7}$$

We extend this definition to involve the quotient of two polynomials and define a **rational expression.**

Rational expressions

A **rational expression** is an expression of the form

$$\frac{P}{Q}$$

where P and Q are polynomials, $Q \neq 0$.

Concept

A rational expression is an expression that can be written as the quotient of two polynomials with the denominator not zero.

For example,

$$\frac{2x}{x+1}, \quad \frac{x^2-2}{x^2-x-6}, \quad \text{and} \quad \frac{x^2+x}{5}$$

are all rational expressions.

Just as a rational number has a numerator and a denominator, so does a rational expression. In the rational expression

$$\frac{x^2-2}{x^2-x-6}$$

the polynomial on the top, $x^2 - 2$, is called the **numerator** and the polynomial on the bottom, $x^2 - x - 6$, is called the **denominator.**

Note Any polynomial is a rational expression since the denominator can be considered to be 1. For example, $x^2 - 2 = \dfrac{x^2-2}{1}$.

Evaluating a rational expression

In chapter 2, we evaluated algebraic expressions by substituting given values for the variables and performing the indicated operations. We follow the same procedure when evaluating rational expressions.

■ ***Example 5–1 A***

Evaluate the following rational expressions for the given value of the variable.

1. $\dfrac{5x-2}{4x+3}$, $x = 2$

$$\frac{5x-2}{4x+3} = \frac{5(2)-2}{4(2)+3}$$ Replace x with 2

$$= \frac{10-2}{8+3}$$ Perform indicated operations using order of operations

$$= \frac{8}{11}$$

2. $\dfrac{x+2}{x^2-3x-10}$, $x = 5$

$$\frac{x+2}{x^2-3x-10} = \frac{5+2}{(5)^2-3(5)-10}$$ Replace x with 5

$$= \frac{7}{25-15-10}$$ Perform indicated operations

$$= \frac{7}{0} \quad \text{(undefined)}$$

Note Anytime zero is in the denominator, the expression is undefined. Therefore, for the rational expression to be defined, x cannot be 5.

3. $\dfrac{x-5}{2x^2+x-1}, x = -\dfrac{1}{2}$

$$\frac{x-5}{2x^2+x-1} = \frac{\left(-\frac{1}{2}\right)-5}{2\left(-\frac{1}{2}\right)^2+\left(-\frac{1}{2}\right)-1}$$ Replace x with $-\frac{1}{2}$

$$= \frac{-\frac{1}{2}-\frac{10}{2}}{\frac{1}{2}-\frac{1}{2}-1}$$ Perform indicated operations

$$= \frac{-\frac{11}{2}}{-1}$$

$$= \frac{11}{2}$$ Multiply numerator and denominator by -1

▶ ***Quick check*** Evaluate: $\dfrac{3y-2}{y+3}$ when $y = 4$ and $\dfrac{y^2-y-1}{y^2-4}$ when $y = -2$ ■

Domain of a rational expression

Notice in example 2, the answer was undefined since division by zero is not defined. The rational expression becomes *meaningless* for those values of the variable for which the denominator equals zero, as with $x = 5$ in example 2. Finding the value(s) of the variable that will make the denominator zero is called *finding the restrictions on the variable(s).* All other values of the variable for which the expression is defined make up the **domain** of the rational expression.

Domain of a rational expression

The set of all replacement values of the variable for which a rational expression is defined determines the **domain** of the rational expression.

In finding the domain of a rational expression, we use the following procedure.

Finding the domain of a rational expression

1. Factor the denominator into a product of prime polynomials, if possible.
2. Set each factor of the denominator containing the variable equal to zero (using the zero product property).
3. Solve the resulting equations. The solutions are the restrictions placed on the variable.

■ Example 5–1 B

Determine the domain of each of the following rational expressions.

1. $\dfrac{a-3}{a-4}$

$a - 4 = 0$ — Set denominator equal to 0

$a = 4$ — Solve equation for a

The restriction is that $a \neq 4$. Domain is all real numbers except 4.

Note We look *only* at the denominator. The value(s) of the variable for which the numerator is zero is of no concern to us.

2. $\dfrac{3x^2}{x^2 - x - 6}$

$\dfrac{3x^2}{x^2 - x - 6} = \dfrac{3x^2}{(x-3)(x+2)}$ — Factor the denominator

$x - 3 = 0$ or $x + 2 = 0$ — Set each factor equal to 0

$x = 3$ $\quad x = -2$ — Solve each equation for x

The restrictions are $x \neq 3$ or $x \neq -2$. Domain is all real numbers except 3 or -2.

3. $\dfrac{x+3}{x^2+4}$

$x^2 + 4$ does not factor. If there is a restriction, it will occur when x^2 is -4. Since x^2 is never negative, the sum $x^2 + 4$ can never be zero and there are no restrictions on the variable. Thus, the domain is the set of all real numbers.

4. $\dfrac{x-3}{x^2-x}$

$\dfrac{x-3}{x^2-x} = \dfrac{x-3}{x(x-1)}$ — Factor the denominator

$x = 0$ or $x - 1 = 0$ — Set each factor equal to 0

$x = 0$ $\quad x = 1$ — Solve each equation for x

The restrictions are $x \neq 0$ or $x \neq 1$. Domain is all real numbers except 0 or 1.

Note In example 4, a common error is to forget to place restrictions on the factor x.

▶ ***Quick check*** Determine the domain: $\dfrac{x+3}{x^2+x-6}$ ■

Mastery points

Can you

- Evaluate a rational expression for a given value of the variable?
- Determine the restrictions on the variable in a rational expression?
- Determine the domain of a rational expression?

Exercise 5–1

Evaluate each given rational expression using the given value of the variable. If the given value is not in the domain of the rational expression, so state. See example 5–1 A.

Examples $\dfrac{3y-2}{y+3}; y=4$ $\qquad$ $\dfrac{y^2+2y-1}{y^2-4}; y=-2$

Solutions

$\dfrac{3(4)-2}{(4)+3}$ Replace y with 4

$\dfrac{12-2}{4+3}$ Perform indicated operations

$\dfrac{10}{7}$

$\dfrac{(-2)^2+2(-2)-1}{(-2)^2-4}$ Replace y with -2

$\dfrac{4-4-1}{4-4}$ Perform indicated operations

$\dfrac{-1}{0}$ (Undefined)

-2 is not in the domain.

1. $\dfrac{x}{3x}; x=2$
2. $\dfrac{2x+1}{5x-3}; x=3$
3. $\dfrac{5a^2+2}{a-1}; a=1$
4. $\dfrac{x^2-1}{x}; x=0$
5. $\dfrac{-4p^2}{2p-3}; p=-1$
6. $\dfrac{-5b^3}{5-2b}; b=-2$
7. $\dfrac{x+9}{x^2+2x-1}; x=4$
8. $\dfrac{2n-3}{3n^2+n-1}; n=-3$
9. $\dfrac{(-2x)^2}{x^2+3x+7}; x=2$
10. $\dfrac{(-x)^3}{2x^2-5}; x=3$
11. $\dfrac{3x^2+2x+1}{4-x-x^2}; x=1$
12. $\dfrac{8-b-2b^2}{4-3b^2}; b=-2$
13. $\dfrac{3-4x}{x^2-x}; x=0$
14. $\dfrac{x^3-2x^2+x}{x-2}; x=\dfrac{1}{2}$
15. $\dfrac{-x+3}{x^2+2x+1}; x=-\dfrac{1}{3}$

Determine the domain of the given rational expression. See example 5–1 B.

Example $\dfrac{x+3}{x^2+x-6}$

Solution

$\dfrac{x+3}{(x+3)(x-2)}$ Factor the denominator

$x+3=0$ or $x-2=0$ Set each factor equal to zero

$x=-3 \qquad x=2$ Solve for x

The restrictions are $x \neq -3$ or $x \neq 2$ and the domain is the set of all real numbers except -3 or 2.

16. $\dfrac{4}{3x}$
17. $\dfrac{5}{4x}$
18. $\dfrac{8}{x-2}$
19. $\dfrac{10}{x-5}$
20. $\dfrac{x}{x+7}$
21. $\dfrac{3x^2}{x+3}$
22. $\dfrac{x+1}{2x-1}$
23. $\dfrac{a+9}{4a-3}$
24. $\dfrac{p-3}{5-2p}$
25. $\dfrac{y+4}{8-3y}$
26. $\dfrac{x+7}{x^2+3x-18}$
27. $\dfrac{8b+1}{b^2-7b+6}$
28. $\dfrac{5s^2+7}{2s^2-s-3}$
29. $\dfrac{8z}{3z^2+2z-8}$
30. $\dfrac{4}{x^2-4}$

31. $\frac{5x}{9x^2 - 4}$

$\boxed{32.}$ $\frac{a - 2}{4a^2 - 16}$

33. $\frac{b + 3}{5b^2 - 45}$

34. $\frac{7x}{3x - 15}$

35. $\frac{5b - 1}{9b - 21}$

36. $\frac{16x}{8x^2 - 18}$

$\boxed{37.}$ $\frac{17q}{3q^2 - 3}$

38. $\frac{23}{x^2 + 1}$

39. $\frac{5x - 3}{2x^2 + 1}$

40. $\frac{2x^2}{x^2 + 9}$

41. In business, the acid-test ratio (A) is given by $A = \frac{C + R}{L}$, where C is the cash on hand, R represents receivables, and L is the current liability. For what value of L is A not defined?

42. In college algebra, the sum S of the terms in some infinite geometric sequence is given by $S = \frac{a}{1 - r}$, where a is the first term and r is the common ratio. For what value of r is S not defined?

43. In physics, the *ideal gas law* of a constant mass of gas is given by $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$, where P_1 = the pressure in the first state, V_1 = the volume in the first state, T_1 = the temperature in Kelvin degrees in the first state, P_2 = the pressure in the second state, V_2 = the volume in the second state, and T_2 = the temperature in Kelvin degrees in the second state. What restrictions on the variables must be placed in this formula?

Review exercises

1. The statement $4(5 + 6) = 4 \cdot 5 + 4 \cdot 6$ demonstrates what property of real numbers? See section 1–8.

2. Simplify the expression $2[3 - 4(1 + 6)]$ by performing the indicated operations. See section 1–8.

3. Evaluate the expression $a[b + (c - d)]$ when $a = 1$, $b = -2$, $c = 3$, and $d = -4$. See section 2–2.

4. Reduce the fraction $\frac{35}{63}$ to lowest terms. See section 1–1.

Factor the following expressions. See section 4–3.

5. $2x^2 - 9x - 5$

6. $4y^2 - 40y + 100$

5–2 ■ Simplifying rational expressions

The fundamental principle of rational expressions

One of the most important procedures we can use when we work with rational expressions is the simplification of the rational expression. To do this, we use a principle called the **fundamental principle of rational expressions.**

Fundamental principle of rational expressions

If P is any polynomial and Q and R are nonzero polynomials, then

$$\frac{PR}{QR} = \frac{P}{Q} \text{ and } \frac{P}{Q} = \frac{PR}{QR}$$

Concept

To change the appearance of a rational expression without changing its value, we may multiply or divide both the numerator and the denominator by the same nonzero polynomial.

This property is based on 1 being the identity element for multiplication. That is,

$$\frac{PR}{QR} = \frac{P}{Q} \cdot \frac{R}{R} = \frac{P}{Q} \cdot 1 = \frac{P}{Q}$$

This property permits us to **reduce** rational expressions to *lowest terms.* A rational expression is *completely reduced* if the greatest factor common to both the numerator and the denominator is 1 or -1. We can see that the key to reducing rational expressions is *finding* and dividing out *factors* that are common to both the numerator and the denominator.

To reduce a rational expression

1. Write the numerator and the denominator in factored form.
2. Divide the numerator and the denominator by all common factors.

Example 5–2 A

Simplify the following rational expressions by reducing to lowest terms. Assume that all denominators are nonzero.

1. $\dfrac{45}{60} = \dfrac{3 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 3 \cdot 5}$ Factor numerator and denominator to prime factors

$= \dfrac{3 \cdot (3 \cdot 5)}{2 \cdot 2 \cdot (3 \cdot 5)}$ Group common factors $(3 \cdot 5)$

$= \dfrac{3}{2 \cdot 2}$ Divide numerator and denominator by $(3 \cdot 5)$

$= \dfrac{3}{4}$ Multiply remaining factors

2. $\dfrac{14x^2}{10x^3} = \dfrac{7 \cdot 2 \cdot x \cdot x}{5 \cdot 2 \cdot x \cdot x \cdot x}$ Factor numerator and denominator

$= \dfrac{7 \cdot (2 \cdot x \cdot x)}{5 \cdot x \cdot (2 \cdot x \cdot x)}$ Group common factors $(2 \cdot x \cdot x)$

$= \dfrac{7}{5x}$ Divide numerator and denominator by common factor $(2 \cdot x \cdot x)$

3. $\dfrac{5a - 15}{4a - 12} = \dfrac{5(a-3)}{4(a-3)}$ Factor numerator and denominator

$= \dfrac{5}{4}$ Divide numerator and denominator by common factor $(a - 3)$

4. $\dfrac{y-7}{y^2 - 49} = \dfrac{y-7}{(y+7)(y-7)}$ Factor denominator

$= \dfrac{1}{y+7}$ Divide numerator and denominator by common factor $(y - 7)$

▶ ***Quick check*** Reduce $\dfrac{25z^4}{15z^5}$ and $\dfrac{a^2 - 36}{a^2 - a - 30}$ to lowest terms. ■

Note This reducing process is often called "cancelling" the common factors. It is important to remember that we are *dividing* both the numerator and the denominator by the same common factor; we are *not* just "crossing out" quantities. This leads us to our next topic.

Common errors when reducing

The fundamental property allows us to divide by common *factors only.* A common error in example 4 is to divide the numerator and the denominator by y and 7. These are *terms* and this cannot be done. For example,

$$\frac{9}{11} = \frac{8+1}{8+3} \neq \frac{\cancel{8}+1}{\cancel{8}+3} = \frac{1}{3}$$

This error can be avoided by always remembering that the *fundamental principle of rational expressions* allows us to *divide* the numerator and the denominator by common factors. y and 7 are *terms* and *not factors.*

Reducing $\frac{a-b}{b-a}$

Consider the rational expression $\frac{x-5}{5-x}$, which does not appear to be reducible by a common factor. However,

$$\begin{aligned} 5-x &= -1(-5+x) && \text{Factor out } -1 \\ &= -1(x-5) && \text{Commute the terms and use the definition of subtraction} \end{aligned}$$

Thus,

$$\begin{aligned} \frac{x-5}{5-x} &= \frac{x-5}{-1(x-5)} \\ &= \frac{1}{-1} && \text{Reduce by common factor } (x-5) \\ &= -1 \end{aligned}$$

In general, for all real numbers a and b, $a \neq b$,

$$\frac{a-b}{b-a} = -1$$

Example 5–2 B

Simplify the following rational expressions by reducing to lowest terms. Assume that no denominator equals zero.

1. $\dfrac{4-x}{x^2-16}$

$$\begin{aligned} &= \frac{4-x}{(x-4)(x+4)} && \text{Completely factor the denominator} \\ &= \frac{4-x}{x-4} \cdot \frac{1}{x+4} && \text{Factor opposites } 4-x \text{ and } x-4 \\ &= -1 \cdot \frac{1}{x+4} && \frac{4-x}{x-4} = -1 \\ &= \frac{-1}{x+4} && \text{Multiply numerator by } -1 \end{aligned}$$

2. $\dfrac{1-x^2}{2x^2+x-3}$

$= \dfrac{(1-x)(1+x)}{(x-1)(2x+3)}$ Completely factor numerator and denominator

$= \dfrac{1-x}{x-1} \cdot \dfrac{1+x}{2x+3}$ Factor opposites $1-x$ and $x-1$

$= -1 \cdot \dfrac{1+x}{2x+3}$ $\dfrac{1-x}{x-1} = -1$

$= \dfrac{-1(1+x)}{2x+3}$ Multiply numerator by -1

$= \dfrac{-1-x}{2x+3}$ or $\dfrac{-x-1}{2x+3}$ Alternative forms of answer

► ***Quick check*** Reduce $\dfrac{16-y^2}{3y^2-11y-4}$ to lowest terms.

Mastery points

Can you

- Reduce a rational expression to lowest terms using the fundamental principle of rational expressions?
- Recognize factors $a-b$ and $b-a$ and use $\dfrac{a-b}{b-a} = -1$?

Exercise 5–2

Simplify the following rational expressions by reducing to lowest terms. Assume that no denominator equals zero. See example 5–2 A.

Examples $\dfrac{25z^4}{15z^5}$

Solutions

$= \dfrac{5 \cdot 5 \cdot z \cdot z \cdot z \cdot z}{5 \cdot 3 \cdot z \cdot z \cdot z \cdot z \cdot z}$ Factor numerator and denominator

$= \dfrac{5 \cdot (5 \cdot z \cdot z \cdot z \cdot z)}{3 \cdot z \cdot (5 \cdot z \cdot z \cdot z \cdot z)}$ Group common factors $(5 \cdot z \cdot z \cdot z \cdot z)$

$= \dfrac{5}{3z}$ Divide numerator and denominator by $(5 \cdot z \cdot z \cdot z \cdot z)$

$\dfrac{a^2-36}{a^2-a-30}$

$= \dfrac{(a+6)(a-6)}{(a+5)(a-6)}$ Factor numerator and denominator

$= \dfrac{a+6}{a+5}$ Divide numerator and denominator by $(a-6)$

1. $\dfrac{54}{72}$
2. $\dfrac{75}{145}$
3. $\dfrac{6x}{15}$
4. $\dfrac{8a}{10}$
5. $\dfrac{16x^2}{12x}$
6. $\dfrac{15b^3}{20b}$
7. $\dfrac{-8x^2}{6x^4}$
8. $\dfrac{3a^6}{-9a^3}$
9. $\dfrac{16a^2b}{20ab^2}$
10. $\dfrac{15a^2x^3}{35ax^2}$
11. $\dfrac{20ab^2c^3}{-4ab^2c^3}$
12. $\dfrac{-72x^4y^3z^2}{9x^4y^3z^2}$
13. $\dfrac{10(x+5)}{8(x+5)}$
14. $\dfrac{24(x-3)}{15(x-3)}$
15. $\dfrac{6(x-2)}{(x+3)(x-2)}$
16. $\dfrac{-8(x+1)}{4(x+1)(x-6)}$

$\boxed{17.}$ $\dfrac{a+b}{a^2-b^2}$ **18.** $\dfrac{x^2-y^2}{x-y}$ **19.** $\dfrac{3m-6}{5m-10}$ **20.** $\dfrac{8b+12}{10b+15}$

21. $\dfrac{3x-3}{6x+6}$ **22.** $\dfrac{6y-6}{8y^2-8}$ **23.** $\dfrac{x^2-9}{x^2+6x+9}$ **24.** $\dfrac{a^2-10a+25}{a^2-25}$

$\boxed{25.}$ $\dfrac{x^2-3x-10}{x^2-x-6}$ **26.** $\dfrac{y^2-y-42}{y^2+12y+36}$ **27.** $\dfrac{2y^2-3y-9}{4y^2-13y+3}$ **28.** $\dfrac{4m^2-15m-4}{8m^2-18m-5}$

29. $\dfrac{x-3}{x^3-27}$ **30.** $\dfrac{x+2}{x^3+8}$ **31.** $\dfrac{a^2-b^2}{a^3+b^3}$ **32.** $\dfrac{x^3+y^3}{x^2-y^2}$

Simplify by reducing to lowest terms. Assume that no denominator is equal to zero. See example 5–2 B.

Example $\dfrac{16-y^2}{3y^2-11y-4}$

Solution

$= \dfrac{(4-y)(4+y)}{(y-4)(3y+1)}$ Factor numerator and denominator

$= \dfrac{4-y}{y-4}\cdot\dfrac{4+y}{3y+1}$

$= -1\cdot\dfrac{4+y}{3y+1}$ $\dfrac{4-y}{y-4} = -1$

$= \dfrac{-1(4+y)}{3y+1}$ Multiply the numerator by -1

$= \dfrac{-4-y}{3y+1}$ or $\dfrac{-y-4}{3y+1}$ Alternative forms of answer

33. $\dfrac{4x-4y}{y-x}$ $\boxed{34.}$ $\dfrac{8b-8a}{a-b}$ **35.** $\dfrac{2x-8}{12-3x}$ **36.** $\dfrac{12a-8b}{10b-15a}$

37. $\dfrac{2y^2-2x^2}{x-y}$ **38.** $\dfrac{3p-3q}{6q^2-6p^2}$ **39.** $\dfrac{(x-y)^2}{y^2-x^2}$ **40.** $\dfrac{a-b}{b^2-a^2}$

41. $\dfrac{n^2-m^2}{(m+n)^2}$ **42.** $\dfrac{p^2-q^2}{q^2-p^2}$ **43.** $\dfrac{4x-4y}{y^2-x^2}$ **44.** $\dfrac{4-y}{2y^2-7y-4}$

$\boxed{45.}$ $\dfrac{x-3}{12-x-x^2}$

Review exercises

1. Write the decimal number 0.000314 in scientific notation. See section 3–5.

2. Given $x = 2$, $y = -3$ and $z = -1$, evaluate the expression $\dfrac{4x-y}{2y+z}$. See sections 2–2 and 5–1.

3. A piece of lumber 16 feet long is to be divided into two pieces so that one piece is 1 foot longer than twice the length of the other piece. Find the lengths of the two pieces of lumber. See section 2–8.

Simplify the following expressions. Assume all denominators are nonzero. Express answers with positive exponents only. See section 3–4.

4. $(5x^{-2}y^3)^0$ **5.** $\dfrac{3xy^2}{3^{-2}xy^{-1}}$ **6.** $(-2x^3y^2)^3$

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5–3 ■ The quotient of two polynomials

In section 3–3, we observed the process of dividing a monomial by a monomial. We shall first review this process before we deal with other types of polynomial division. Recall the quotient property for division of expressions that have like bases, $a^m \div a^n = a^{m-n}$, $a \neq 0$.

■ Example 5–3 A

Find the indicated quotients. Assume all variables are nonzero.

1. $x^7 \div x^4 = x^{7-4}$ — Subtract exponents when dividing
$= x^3$

2. $\dfrac{2^2a^5}{2a^3} = 2^{2-1}a^{5-3}$ — Divide like bases by subtracting exponents
$= 2^1a^2$ — Perform indicated subtractions
$= 2a^2$ — $2^1 = 2$

▶ ***Quick check*** Find the quotient of $\dfrac{5^3x^4}{5x}$ ■

Division of a polynomial by a monomial

Consider the indicated division.

$$\frac{3x^3 - 9x^2 + 15x}{3x}$$

To perform this division, we use a principle of fractions.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}\ (c \neq 0)$$

By reversing this equation, the principle can be used to divide a polynomial by a monomial.

Division of a polynomial by a monomial

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}\ (c \neq 0)$$

Concept

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

■ Example 5–3 B

Find the indicated quotients. Assume all denominators are nonzero.

1. $\dfrac{3x^3 - 9x^2 + 15x}{3x} = \dfrac{3x^3}{3x} - \dfrac{9x^2}{3x} + \dfrac{15x}{3x}$ — Divide each term of the numerator by the monomial denominator
$= \dfrac{3}{3}x^{3-1} - \dfrac{9}{3}x^{2-1} + \dfrac{15}{3}x^{1-1}$ — Quotient property of exponents
$= 1 \cdot x^2 - 3 \cdot x^1 + 5 \cdot x^0$ — Subtract and divide as indicated
$= x^2 - 3x + 5 \cdot 1$ — $x^0 = 1$
$= x^2 - 3x + 5$

2. $$\frac{8a^4 + 4a^2 - 12a}{4a} = \frac{8a^4}{4a} + \frac{4a^2}{4a} - \frac{12a}{4a}$$ Divide each term of numerator by the monomial denominator

$$= 2a^3 + a - 3$$ Simplify each term by reducing

3. $$\frac{5a^7 + 15a^5 - 10a}{5a^2} = \frac{5a^7}{5a^2} + \frac{15a^5}{5a^2} - \frac{10a}{5a^2}$$ Divide each term of numerator by the monomial denominator

$$= a^5 + 3a^3 - \frac{2}{a}$$ Simplify each term by reducing

Recall that we can check our division by

(quotient)(divisor) = dividend

In example 2,

$$(2a^3 + a - 3)(4a) = 2a^3 \cdot 4a + a \cdot 4a - 3 \cdot 4a$$ Distributive property

$$= 8a^4 + 4a^2 - 12a$$ Dividend

▶ ***Quick check*** Find the quotient: $\dfrac{16x^5 + 20x^3 - 4x^2}{4x^2}$ ■

Note A common error in this type of problem is demonstrated in the following example.

$$\frac{x^3 + x^2}{x^2} \neq \frac{x^3 + 1}{1}$$

It is tempting to simply "cancel" the x^2 in the numerator with the x^2 in the denominator, but the correct procedure would be

$$\frac{x^3 + x^2}{x^2} = \frac{x^3}{x^2} + \frac{x^2}{x^2} = x + 1$$

Remember that the entire numerator and the entire denominator must be divided by the same quantity. In the example, only *part* of the numerator was divided by x^2. *Only factors* may be divided by factors.

Division of a polynomial by a polynomial

Consider a quotient in which the divisor is not a monomial. For example,

$$\frac{y^2 - y - 2}{y - 2} \begin{array}{l} \longleftarrow \text{Dividend} \\ \longleftarrow \text{Divisor} \end{array}$$

which involves the division of a trinomial by a binomial. We handle this just like long division with numbers. Set it up in the form

$$y - 2\overline{)y^2 - y - 2}$$

Note The divisor and dividend must be arranged in descending powers of one variable with zeros inserted to hold the position of any missing term.

The following table demonstrates writing a polynomial in descending powers of the variable and inserting zeros to hold the position of missing terms.

Dividend	**Dividend arranged in descending powers**
$x^3 + 2x + 3x^4 + 4x^2 - 1$	$3x^4 + x^3 + 4x^2 + 2x - 1$
$x^3 + x - 9$	$x^3 + 0x^2 + x - 9$
$x^4 - 1$	$x^4 + 0x^3 + 0x^2 + 0x - 1$

The method for dividing polynomials is similar to the long division used in dividing whole numbers. To demonstrate this, we divide 972 by 36 step-by-step as we divide $(y^2 - y - 2)$ by $(y - 2)$.

$$36\overline{)972} \qquad\qquad y - 2\overline{)y^2 - y - 2}$$

Step 1 Divide 36 into 97, which goes 2 times. Place 2 over 7 in the dividend.

$$\begin{array}{r} 2 \\ 36\overline{)972} \end{array}$$

Divide y into y^2, which goes y times. Place y over y in the dividend.

$$\begin{array}{r} y\phantom{{}-y-2} \\ y - 2\overline{)y^2 - y - 2} \end{array}$$

Step 2 Multiply 2 times 36, place 72 below 97 in the dividend.

$$\begin{array}{r} 2 \\ 36\overline{)972} \\ \underline{72} \end{array}$$

Multiply y times $(y - 2)$, place $y^2 - 2y$ below $y^2 - y$ in the dividend.

$$\begin{array}{r} y\phantom{{}-2} \\ y - 2\overline{)y^2 - y - 2} \\ \underline{y^2 - 2y}\phantom{{}-2} \end{array}$$

Step 3 Subtract 72 from 97. The difference is 25.

$$\begin{array}{r} 2 \\ 36\overline{)972} \\ (-)\underline{72} \\ 25 \end{array}$$

Subtract $y^2 - 2y$ from $y^2 - y$.
$(y^2 - y) - (y^2 - 2y) =$
$y^2 - y - y^2 + 2y = y$

$$\begin{array}{r} y\phantom{{}-2} \\ y - 2\overline{)y^2 - y - 2} \\ (-)\underline{y^2 - 2y}\phantom{{}-2} \\ y\phantom{{}-2} \end{array}$$

Change signs and add

Step 4 Bring down the next digit of the dividend, 2.

$$\begin{array}{r} 2 \\ 36\overline{)972} \\ \underline{72} \\ 252 \end{array}$$

Bring down the next term of the dividend, -2.

$$\begin{array}{r} y\phantom{{}-2} \\ y - 2\overline{)y^2 - y - 2} \\ \underline{y^2 - 2y}\phantom{{}-2} \\ y - 2 \end{array}$$

Step 5 Divide 36 into 252, which goes 7 times. Place 7 over 2 in the dividend.

$$\begin{array}{r} 27 \\ 36\overline{)972} \\ \underline{72} \\ 252 \end{array}$$

Divide y into y, which goes 1 time. Place 1 over 2 in the dividend with a plus sign between y and 1.

$$\begin{array}{r} y + 1 \\ y - 2\overline{)y^2 - y - 2} \\ \underline{y^2 - 2y}\phantom{{}-2} \\ y - 2 \end{array}$$

Step 6 Multiply 7 times 36, which is 252. Place this product below 252 at the bottom.

$$\begin{array}{r} 27 \\ 36\overline{)972} \\ \underline{72} \\ 252 \\ (-)\underline{252} \\ 0 \end{array}$$

Multiply 1 times $(y - 2)$, which is $y - 2$. Place this below $y - 2$ at the bottom.

$$\begin{array}{r} y + 1 \\ y - 2\overline{)y^2 - y - 2} \\ \underline{y^2 - 2y} \\ y - 2 \\ (-)\underline{y - 2} \\ 0 \end{array}$$

Change signs and add

Step 7 Subtract $252 - 252 = 0$. There is no remainder.

$972 \div 36 = 27$

Subtract $(y - 2) - (y - 2) = 0$.

$(y^2 - y - 2) \div (y - 2)$
$= y + 1$

Step 8 Check your division by multiplying the quotient by the divisor to see if you get the original dividend.

$27 \cdot 36 = 972$ $\qquad$ $(y + 1)(y - 2) = y^2 - y - 2$

Note A common error is committed when we subtract polynomials as we did in step 3. Remember, to subtract two polynomials, *change the signs of the second polynomial and then add.*

$$\begin{array}{rcr} y^2 - y & \rightarrow & y^2 - y \\ (-)\ \underline{y^2 - 2y} & \rightarrow & \underline{-y^2 + 2y} \\ & & 0 + y = y \end{array}$$

The large majority of errors in this type of problem occur when polynomials are subtracted.

■ Example 5–3 C

Find the indicated quotient and check the answer.

1. $\dfrac{x^2 + 3x - 4}{x + 4}$

$$\begin{array}{r} x - 1 \\ x + 4\overline{)x^2 + 3x - 4} \\ \underline{x^2 + 4x} \\ -x - 4 \\ \underline{-x - 4} \\ 0 \end{array}$$

$x(x + 4) = x^2 + 4x$
Subtract to get $-x$ and bring down -4
$-1(x + 4) = -x - 4$
Subtract to get 0

Therefore, $\dfrac{x^2 + 3x - 4}{x + 4} = x - 1$.

Check: $(x - 1)(x + 4) = x^2 + 4x - x - 4$
$\qquad\qquad\qquad\qquad = x^2 + 3x - 4$

If we still have a remainder after "bringing down" all of the terms of the dividend, handle it as follows:

2. $\dfrac{a^2 + 5a + 6}{a - 2}$

$$\begin{array}{r} a + 7 \\ a - 2\overline{)a^2 + 5a + 6} \\ \underline{a^2 - 2a} \\ 7a + 6 \\ \underline{7a - 14} \\ 20 \end{array}$$

$a(a - 2) = a^2 - 2a$
Subtract to get $7a$. Bring down 6
$7(a - 2) = 7a - 14$
$(7a + 6) - (7a - 14) = 7a + 6 - 7a + 14 = 20$

Hence, $\dfrac{a^2 + 5a + 6}{a - 2} = a + 7 + \dfrac{20}{a - 2}$, where the remainder 20 is placed over the divisor $a - 2$.

To check our answer, we add the remainder of 20 to the product of $(a + 7)$ and $(a - 2)$.

$$\begin{aligned}(a + 7)(a - 2) + (20) &= a^2 - 2a + 7a - 14 + (20)\\ &= a^2 + 5a + 6\end{aligned}$$

3. $\dfrac{x^3 - x + 2}{x - 3}$

Note that there is no term in the dividend that contains x^2. The division will be easier to perform if the term $0x^2$ is inserted as a placeholder so that all powers of the variable x are present in descending order. Thus, we have

$$\frac{x^3 + 0x^2 - x + 2}{x - 3}$$

and the value of the dividend has not been changed since we have added $0x^2$, which is 0. Therefore, to perform the division, we get

$$\begin{array}{r} x^2 + 3x + 8 \\ x - 3\overline{)x^3 + 0x^2 - x + 2} \\ (-)\underline{x^3 - 3x^2} \qquad\qquad \\ 3x^2 - x \qquad \\ (-)\underline{3x^2 - 9x} \qquad \\ 8x + 2 \\ (-)\underline{8x - 24} \\ 26 \end{array}$$

$$\frac{x^3 - x + 2}{x - 3} = x^2 + 3x + 8 + \frac{26}{x - 3}$$

Check: $(x^2 + 3x + 8)(x - 3) + (26)$
$= x^3 - 3x^2 + 3x^2 - 9x + 8x - 24 + 26$
$= x^3 - x + 2$

Note When performing division, always count the number of terms in the denominator. If there is only one term, *do not use long division.*

► ***Quick check*** Find the quotient: $\dfrac{6x^2 - 7x - 3}{2x - 3}$

Mastery points

Can you

- Divide a monomial by a monomial?
- Divide a polynomial by a monomial?
- Divide a polynomial by a polynomial?
- Check the answer?

Exercise 5–3

Perform the indicated divisions and check the answers. See example 5–3 A.

Example $\dfrac{5^3x^4}{5x}$

Solution
$= 5^{3-1}x^{4-1}$ Divide like bases by subtracting exponents
$= 5^2x^3$ Perform subtractions
$= 25x^3$ $5^2 = 25$

1. $\dfrac{8x^3}{2x}$
2. $\dfrac{-15x^5}{3x^2}$
3. $\dfrac{-65x^4y^2z}{13xy}$
4. $\dfrac{-28a^3b}{-7ab}$
5. $\dfrac{3(a-b)^2}{a-b}$
6. $\dfrac{5(x+y)^3}{x+y}$
7. $\dfrac{6a^2(b-c)^2}{3a(b-c)}$
8. $\dfrac{-10x^3(y-z)^3}{2x^2(y-z)}$
9. $\dfrac{12a^3b^2c(x+y)^3}{-3abc(x+y)^2}$

See example 5–3 B.

Example $\dfrac{16x^5 + 20x^3 - 4x^2}{4x^2}$

Solution
$= \dfrac{16x^5}{4x^2} + \dfrac{20x^3}{4x^2} - \dfrac{4x^2}{4x^2}$ Divide denominator into each term of numerator
$= 4x^3 + 5x - 1$ Divide constants and apply properties of exponents

10. $\dfrac{6x-9}{3}$
11. $\dfrac{24a^2 - 12a}{-6}$
12. $\dfrac{bx^2 - bx}{bx}$
13. $\dfrac{a^3 - 3a^2 + 2a}{a}$
14. $\dfrac{12x^3 - 8x^2 + 3x}{4x}$
15. $\dfrac{15a^3 - 9a^2 + 12a - 6}{3a}$
16. $\dfrac{13a - a^2b^2 + a^2b}{a^2b}$
17. $\dfrac{x^2y - xy^2 - 2xy^3}{-xy}$
18. $\dfrac{14a^2b^3 - 21a^2b^2 - 28ab}{7ab}$
19. $\dfrac{30x^3y^4 + 21x^2y^2 - 18x^2y^4}{3x^2y^2}$
20. $\dfrac{-21m^2n^5 + 35m^3n^2 - 14m^2n^2}{-7m^2n^2}$
21. $\dfrac{a(b-1) - c(b-1)}{b-1}$
22. $\dfrac{a(x-y) - b(x-y)}{x-y}$

See example 5–3 C.

Example $\dfrac{6x^2 - 7x - 3}{2x - 3}$

Solution

$$\begin{array}{r l}
 & 3x + 1 \\
2x - 3 \,\big)\!\! & \overline{6x^2 - 7x - 3} \\
 & \underline{(-)6x^2 - 9x} \\
 & 2x - 3 \\
 & \underline{(-)2x - 3} \\
 & 0
\end{array}$$

$3x(2x-3) = 6x^2 - 9x$
Subtract to get $2x$ and bring down -3
$1(2x-3) = 2x - 3$
$(2x-3) - (2x-3) = 2x - 3 - 2x + 3 = 0$

$$\frac{6x^2 - 7x - 3}{2x - 3} = 3x + 1$$

The check is left to the student.

23. $\dfrac{a^2 + 7a + 10}{a - 2}$

24. $\dfrac{x^2 + 8x + 15}{x + 5}$

25. $\dfrac{a^2 + 5a + 10}{a + 3}$

26. $\dfrac{x^2 - x - 72}{x + 8}$

27. $(a^2 + 6a + 10) \div (a + 3)$

28. $(4a^2 + 1 + 4a) \div (2a + 1)$

29. $(9a^2 - 24a + 12) \div (3a - 4)$

30. $(27a^3 - 1) \div (3a - 1)$

31. $(x^3 - 8) \div (x - 2)$

32. $(x^4 - 14) \div (x - 2)$

33. $\dfrac{x^3 + 4x^2 + 7x + 6}{x + 2}$

34. $\dfrac{2a^3 - 3a^2 - 13a + 12}{a - 5}$

35. $\dfrac{b^3 + 6b^2 + 7b - 8}{b - 1}$

36. $\dfrac{6x^4 - x^3 - 2x^2 - 7x - 19}{2x - 3}$

37. $(15a^2 + 28a - 32) \div (5a - 4)$

38. $(x^4 - 2x^3 + 4x^2 - x + 3) \div (x^2 - x + 4)$

39. $(x^4 + 3x^3 - 6x^2 + 3x - 8) \div (x^2 + 3x - 5)$

40. $(y^4 + 2y^3 - 4y + 2) \div (y^2 - y + 1)$

41. $(y^4 + 2y - 3) \div (y^2 + 2y - 5)$

42. A contractor uses the expression $x^2 + 6x + 8$ to represent the area of the floor of a room. If she decides that the length of the room will be represented by $x + 4$, what will the width of the room be in terms of x?

43. An electrician uses the expression $4x^2 + 11x + 6$ to determine the amount of wire to order when wiring a house. If the expression comes from multiplying the number of rooms times the number of outlets and he knows the number of rooms to be $x + 2$, find the number of outlets in terms of x.

44. What polynomial when divided by $3x - 2$ yields the quotient $2x^2 + 3x - 5$?

45. What polynomial when divided by $-2x + 5$ yields the quotient $3x^3 - 2x + 6$?

Review exercises

1. Find the solution set of the quadratic equation $4y^2 + 9y + 2 = 0$. See section 4–7.

2. The area of a rectangle is 42 square feet. If the length is 1 foot longer than the width, what are the dimensions of the rectangle? See section 4–8.

Find the following products. See section 3–2.

3. $(4x - 3)^2$

4. $(x + 2)(x^2 + x - 1)$

5. $(5y - 1)(5y + 1)$

6. Reduce $\dfrac{3y^2 - 5y - 2}{2y^2 - y - 6}$ to lowest terms. See section 5–2.

5–4 ■ Ratio and proportion

A ratio

We learned the fraction $\frac{a}{b}$ represents the indicated quotient of a divided by b. A **ratio** compares two numbers, or quantities, in the same way.

Ratio

A ratio is the comparison of two numbers (or quantities) by division.

The ratio of the number a to the number b is written

$$a \text{ to } b, \quad \frac{a}{b}, \quad \text{or} \quad a : b$$

We read $a : b$ as "the ratio of a to b," where a and b are called the *terms* of the ratio. The first number given is always the numerator and the second number is the denominator of the fraction representing the ratio.

■ Example 5–4 A

Write each ratio statement in the forms $a : b$ and $\frac{a}{b}$ reduced to lowest terms.

1. The ratio of 3 to 4

$3 : 4$ — Form $a : b$

$\frac{3}{4}$ — Written as a fraction

2. The ratio of 15 to 9

$15 : 9$ or $5 : 3$ — Form $a : b$ (divide 15 and 9 by the common factor 3)

$\frac{15}{9} = \frac{5}{3}$ — Written as a fraction reduced to lowest terms

3. The ratio of $2\frac{1}{2}$ to $3\frac{1}{4}$

We first write each mixed number as an improper fraction.

$2\frac{1}{2} = \frac{5}{2}$ and $3\frac{1}{4} = \frac{13}{4}$

$$\frac{2\frac{1}{2}}{3\frac{1}{4}} = \frac{5}{2} \div \frac{13}{4} = \frac{5}{2} \cdot \frac{4}{13} = \frac{10}{13}$$

Therefore, the ratio $2\frac{1}{2} : 3\frac{1}{4}$ becomes $10 : 13$ or $\frac{10}{13}$.

▶ ***Quick check*** Write the ratio of 18 to 21 in the forms $a : b$ and $\frac{a}{b}$ reduced to lowest terms.

If the quantities have the same unit of measure, the ratio will be expressed by a fraction without any unit designation required.

4. 45 minutes to 60 minutes

$$45 \text{ min} : 60 \text{ min} = \frac{45 \text{ min}}{60 \text{ min}} \qquad \text{Write as fraction } \frac{a}{b}$$

$$= \frac{3}{4} \text{ or } 3 : 4 \qquad \text{Reduce by dividing each term by 15}$$

When the compared quantities are not of the same unit of measure but *can be stated* in the same unit, it may be desirable to do so. The ratio again becomes only a fraction, as in example 4.

5. 3 feet to 4 inches

Since 1 ft = 12 in., then 3 ft = 36 in., we have

$$36 \text{ in.} : 4 \text{ in.} = \frac{36 \text{ in.}}{4 \text{ in.}} \qquad \text{Write as fraction } \frac{a}{b}$$

$$= \frac{9}{1} \text{ or } 9 : 1 \qquad \text{Reduce by dividing each term by 4}$$

Note We reduced to $\frac{9}{1}$ to demonstrate the comparison that is present.

6. 35 cents to 4 dollars

Since there are 400 cents in 4 dollars, we have

$$35 \text{ cents} : 400 \text{ cents} = \frac{35 \text{ cents}}{400 \text{ cents}} \qquad \text{Write as a fraction } \frac{a}{b}$$

$$= \frac{7}{80} \text{ or } 7 : 80 \qquad \text{Reduce by dividing each term by 5}$$

Note When we change to a common unit of measure, it is easiest to change to the *smaller* unit of measure, as we did in the previous examples. Changing to the larger unit of measure usually involves fractions that are more difficult to reduce.

▶ ***Quick check*** Write the ratio of 16 minutes to 2 hours in the forms $a : b$ and $\frac{a}{b}$ reduced to lowest terms. ■

Ratios are used to indicate relationships in many areas of the physical world.

1. The geographer makes maps and prints to scale, 20 miles to 1 inch.
2. The physicist measures air pressure and uses force per unit of area.

   $$14.7 \text{ lb/in.}^2 = \frac{14.7 \text{ lb}}{1 \text{ in.}^2}$$

3. The auto mechanic interprets engine specifications by compression ratio, 9 to 1 or 9 : 1.
4. The machinist is concerned with gear ratio, 2 to 1 or 2 : 1.

When you compare two measurable quantities by ratio, it is not necessary for them to have the same unit of measure. If the units are not the same, you *must include the units* when you are expressing the ratio. These ratios represent rates of change.

Example 5–4 B

Express the following as a ratio in lowest terms.

1. 50 miles to 1 inch

$$50 \text{ mi} : 1 \text{ in.} = \frac{50 \text{ mi}}{1 \text{ in.}} \text{ or } 50 \text{ miles per inch}$$

2. 350 miles to 7 hours

$$350 \text{ mi} : 7 \text{ hr} = \frac{350 \text{ mi}}{7 \text{ hr}}$$ Write as a fraction $\frac{a}{b}$

$$= \frac{50 \text{ mi}}{1 \text{ hr}} \text{ (stated 50 miles per hour)}$$ Reduce by dividing each term by 7

▶ ***Quick check*** Express 64 pounds to 8 square inches as a ratio in lowest terms.

A proportion

A **proportion** establishes a relationship between two ratios.

Definition of a proportion

A **proportion** is a statement of equality of two ratios.

Given the ratios a to b and c to d,

$$\frac{a}{b} = \frac{c}{d} \quad \text{or} \quad a : b = c : d$$

is a proportion. We read the statement $a : b = c : d$ "a is to b as c is to d." The numbers a, b, c, and d are called the *terms* of the proportion.

Given the proportion $\frac{a}{b} = \frac{c}{d}$,

$$bd \cdot \frac{a}{b} = bd \cdot \frac{c}{d}$$ Multiply each member by bd

$$ad = bc$$ Reduce by b on the left and by d on the right

Property of proportions

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$ (b, $d \neq 0$)

Note The products ad and bc are found by multiplying diagonally.

$$\frac{a}{b} \times \frac{c}{d} \qquad \begin{matrix} bc \\ ad \end{matrix}$$

This process is frequently called *cross-multiplying,* especially by persons in applied fields, and ad and bc are called the *cross products.*

■ **Example 5–4 C**

Determine if the following statements form a proportion.

1. $\frac{3}{5} = \frac{12}{20}$

Using the property of proportions, we obtain

$5 \cdot 12 = 60 \quad \text{and} \quad 3 \cdot 20 = 60$

The cross products are both 60, so we have a proportion.

2. $\frac{5}{6} = \frac{16}{18}$

Using the property of proportions, we obtain

$6 \cdot 16 = 96 \quad \text{and} \quad 5 \cdot 18 = 90$

The cross products are not the same so we do not have a proportion. ■

We use the property of proportions to find the unknown term of a proportion if three of the four terms are known.

■ **Example 5–4 D**

Find the unknown term of the given proportion. Check your solution.

1. $\frac{x}{8} = \frac{16}{64}$

$$\begin{aligned} 64 \cdot x &= 8 \cdot 16 && \text{Property of proportions} \\ 64x &= 128 && \text{Multiply as indicated} \\ x &= 2 && \text{Divide each member by 64} \end{aligned}$$

Check: $\frac{2}{8} = \frac{16}{64}$ Then

$$\begin{aligned} 2 \cdot 64 &= 8 \cdot 16 \\ 128 &= 128 \end{aligned}$$

2. $\frac{49}{y} = \frac{35}{5}$

$$\begin{aligned} 49 \cdot 5 &= 35 \cdot y && \text{Property of proportions} \\ 35y &= 245 && \text{Multiply as indicated} \\ y &= 7 && \text{Divide each member by 35} \end{aligned}$$

Check: $\frac{49}{7} = \frac{35}{5}$ Then

$$\begin{aligned} 5 \cdot 49 &= 7 \cdot 35 \\ 245 &= 245 \end{aligned}$$

▶ ***Quick check*** Find the value of z in the proportion $\frac{72}{z} = \frac{30}{6}$. ■

Proportions are used in solving many applied problems. Consider the following examples.

■ Example 5–4 E

Set up a proportion for each problem and solve.

1. Two gears are in the ratio of 4 : 5. If the smaller gear has 32 teeth, how many teeth are there in the larger gear?

   Let x = the number of teeth in the larger gear. Set up a proportion: one ratio involving 4 and the number of teeth in the smaller gear and the other involving 5 and the number of teeth in the larger gear. Corresponding numbers must be in the numerator and the denominator.

   $$\frac{4}{32} = \frac{5}{x} \quad \text{Set up a proportion}$$
   $$4 \cdot x = 32 \cdot 5 \quad \text{Property of proportions}$$
   $$4x = 160 \quad \text{Multiply as indicated}$$
   $$x = 40 \quad \text{Divide each member by 4}$$

   Thus, there are 40 teeth in the larger gear.

2. On a map, 1 inch represents 6 miles. How many inches are needed to represent 28 miles?

   Let x = the number of inches representing 28 miles. Now, 1 inch is to 6 miles as x inches is to 28 miles.

   $$\frac{1 \text{ in.}}{6 \text{ mi}} = \frac{x \text{ in.}}{28 \text{ mi}} \quad \text{Set up a proportion}$$
   $$6 \cdot x = 1 \cdot 28 \quad \text{Property of proportions}$$
   $$x = \frac{1 \cdot 28}{6} = \frac{28}{6} \quad \text{Divide each member by 6}$$
   $$= \frac{14}{3} \text{ or } 4\frac{2}{3} \quad \text{Reduce to lowest terms}$$

   Therefore, 28 miles are represented by $4\frac{2}{3}$ inches on the map.

Note In example 2, **the same units of measure are in the numerator of the ratios and the same units of measure are in the denominators.** That is, we placed inches in the numerator and miles in the denominator of each ratio. This step is important in setting up the proportion you will use to solve for the unknown.

3. Cheryl set aside \$20 per week for her savings program when her salary was \$200 per week. If her salary is now \$250 per week, how much should she set aside for her weekly savings to be proportional to what she saved before?

   Let x = the amount to be set aside when Cheryl earns \$250 per week. Then, \$20 is to \$200 as x is to \$250.

   $$\frac{20}{200} = \frac{x}{250} \quad \text{Set up a proportion}$$
   $$200 \cdot x = 20 \cdot 250 \quad \text{Property of proportions}$$
   $$200x = 5{,}000 \quad \text{Multiply as indicated}$$
   $$x = \frac{5{,}000}{200} \quad \text{Divide each member by 200}$$
   $$x = 25$$

   Cheryl should set aside \$25 when making \$250 per week.

▶ ***Quick check*** On a map, 1 inch represents 9 miles. How many inches are needed to represent 42 miles?

Mastery points

Can you

- Write ratios?
- Reduce ratios?
- Set up proportions?
- Solve proportions for the unknown?
- Set up proportions to solve problems?

Exercise 5–4

Express the given ratios in two forms, $\frac{a}{b}$ and $a : b$, reduced to lowest terms. See example 5–4 A.

Example The ratio of 18 to 21

Solution $18 : 21$ or $6 : 7$ — Form $a : b$ (reduce by dividing each term by 3)

$\frac{18}{21}$ or $\frac{6}{7}$ — Write as a fraction reduced to lowest terms

1. 12 to 7 **2.** 8 to 19 **3.** 7 to 42 **4.** 32 to 60

5. 16 to 6 **6.** 24 to 9 **7.** 15 to 5 **8.** 90 to 40

$\boxed{9.}$ 8 to $\frac{3}{4}$ **10.** 12 to $1\frac{3}{4}$ **11.** $2\frac{1}{2}$ to 10 **12.** $5\frac{5}{6}$ to 3

$\boxed{13.}$ $\frac{5}{6}$ to $\frac{2}{3}$ **14.** $\frac{7}{5}$ to $\frac{3}{4}$ **15.** $3\frac{1}{4}$ to $2\frac{2}{3}$ **16.** $3\frac{4}{5}$ to $4\frac{1}{10}$

$\boxed{17.}$ 4.2 to 2.1 **18.** 2.4 to 6.0 **19.** 1.02 to 2.38 **20.** 3.06 to 2.55

Find the indicated ratios reduced to lowest terms expressed in two forms. See examples 5–4 A and B.

Example 16 minutes to 2 hours

Solution Since 2 hours = 120 minutes (1 hour = 60 minutes),

$$\begin{aligned} 16 \text{ min} : 2 \text{ hr} &= 16 \text{ min} : 120 \text{ min} && \text{Replace 2 hr with 120 min} \\ &= 16 : 120 && \text{Eliminate unit of measure} \\ &= 2 : 15 \text{ or } \frac{2}{15} && \text{Reduce to lowest terms (Divide by 8)} \end{aligned}$$

The ratio of 16 min to 2 hr is $2 : 15$ or $\frac{2}{15}$.

Example 64 pounds to 8 square inches

Solution

$$\begin{aligned} 64 \text{ lb to } 8 \text{ sq in.} &= \frac{64 \text{ lb}}{8 \text{ sq in.}} && \text{Write ratio as a fraction} \\ &= \frac{8 \text{ lb}}{1 \text{ sq in.}} && \text{Divide by 8 to reduce to lowest terms} \\ &= 8 \text{ lb/sq in.} && \text{Write as a rate} \end{aligned}$$

21. 6 in. to 14 in.
22. 4 ft to 18 ft
23. 25 cm to 10 cm
24. 35 lb to 5 lb
25. 36 km to 24 km
26. 48 lb to 16 lb
27. 15 in. to 3 ft
28. 10 ft to 4 yd
29. \$3 to 35¢
30. 5 days to 15 weeks
31. 30 min to 13 hr
32. 16 lb to 8 oz
33. 48 lb to 24 ft^3
34. 50 cm to 5 $in.^3$
35. 16 grams to 2 cm^3
36. 300 mi to 10 gal
37. 1,020 mi to 17 hr
38. 105 kg to 35 m^3

Solve the following applied problems.

39. The *output* in horsepower is the useful energy delivered *by an engine* and the *input* in horsepower is the amount of energy delivered *to an engine*. The *mechanical efficiency* of the engine is given by the ratio

$$\text{mechanical efficiency} = \frac{\text{output}}{\text{input}}$$

Find the mechanical efficiency of an engine rated to deliver 425 horsepower (input) when it delivers only 375 horsepower.

40. The *pitch* of a roof is the ratio of the *rise* of a rafter to the *span* of the roof.

$$\text{pitch} = \frac{\text{rise of rafter}}{\text{span of roof}}$$

Find the pitch if the roof rises 7 feet in a span of 21 feet.

41. The smaller of two belted pulleys makes 240 revolutions per minute and the larger one makes 100 revolutions per minute. What is the ratio of the speed of the larger pulley to the smaller pulley? Of the smaller pulley to the larger pulley?

42. The *mechanical advantage* (*MA*) of a machine is given by the ratio

$$\text{mechanical advantage } (MA) = \frac{\text{resistance to effort (output)}}{\text{input effort}}$$

If a machine has an effort of 30 pounds that results in a resistance of 120 pounds, what is the mechanical advantage of the machine?

43. The mechanical advantage of a hydraulic press can be given by the ratio

$$\text{mechanical advantage} = \frac{\text{area of the large piston}}{\text{area of the small piston}}$$

If the large piston has area 32 square centimeters and the small piston has area 12 square centimeters, find the mechanical advantage of the press.

44. Tool steel may be worked at a cutting speed of 20 feet per minute in a lathe, and cast iron may be worked at a cutting speed of 45 feet per minute. What is the ratio of the cutting speed of tool steel to cast iron?

45. An automobile engine is rated at 350 horsepower. When the engine is tested, it produces only 325 horsepower. What is the mechanical efficiency of the engine? (Refer to exercise 39.)

46. An electric motor uses 10 volts of electricity to produce an equivalent output of 8 volts. What is the mechanical efficiency of the motor? (Refer to exercise 39.)

47. A particular stock costing \$63 paid an earnings of \$6. What is the cost : earnings ratio?

48. A room is 24 feet long and 18 feet wide. What is the ratio of its length to its width?

49. A cement block weighs 120 pounds and a steel block weighs 1,860 pounds. What is the ratio of the weight of the steel block to the weight of the cement block?

50. A mathematics class contains 32 male students and 10 female students. What is the ratio of the male students to the female students?

51. A doctor having earnings of \$60,000 in a given year paid income taxes of \$8,400. What is the ratio of taxes to income?

52. The stress caused by a heavy load is defined by the ratio

$$\text{stress} = \frac{\text{distorting force } (F)}{\text{area } (A)}$$

measured in lb/in.^2 Find the stress of a force of a 4,400-pound load on an area of 1,200 square inches.

53. Power is defined by the ratio of the work done (F) to the time taken (t). Find the power if 42 ft-lb of work is done in 6 seconds, in ft-lb/sec.

54. The magnification (M) of an object by a lens is given by the ratio

$$M = \frac{q}{p}$$

where q = the image distance and p = the object distance from the lens. Find the magnification of an object whose image distance is 27 feet and object distance is 12 feet.

Find the value of the unknown that makes the proportion true. See example 5–4 D.

Example Find the value of z in the proportion $\frac{72}{z} = \frac{30}{6}$.

Solution

$$\frac{72}{z} = \frac{30}{6}$$

$$30 \cdot z = 72 \cdot 6 \qquad \text{Property of proportions}$$

$$30z = 432 \qquad \text{Perform indicated multiplication}$$

$$z = \frac{432}{30} \qquad \text{Divide by 30}$$

$$= \frac{72}{5} \text{ or } 14\frac{2}{5} \qquad \text{Reduce to lowest terms}$$

Thus, $z = 14\frac{2}{5}$ makes the equation a proportion.

$\boxed{55.}$ $\frac{9}{x} = \frac{36}{5}$

56. $\frac{y}{7} = \frac{30}{42}$

57. $\frac{5}{9} = \frac{p}{20}$

58. $\frac{14}{10} = \frac{21}{z}$

$\boxed{59.}$ $6 : 15 = x : 8$

60. $R : 12 = 15 : 100$

61. $1\frac{1}{2} : a = 4\frac{3}{4} : 2$

$\boxed{62.}$ $\frac{3}{4} : 4 = \frac{1}{2} : b$

63. $1.2 : x = 3.6 : 9$

64. $4.5 : 3 = y : 2$

65. $\dfrac{3\frac{1}{4}}{5} = \dfrac{2\frac{1}{2}}{a}$

66. $\frac{2.4}{4.2} = \frac{b}{2.1}$

Solve the following problems by first choosing a letter to represent the unknown and then setting up the proper proportion. See example 5–4 E.

Example On a map, 1 inch represents 9 miles. How many inches are needed to represent 42 miles?

Solution Let x = the number of inches representing 42 miles.
Then, 1 inch is to 9 miles as x inches is to 42 miles.

$$\frac{1 \text{ inch}}{9 \text{ miles}} = \frac{x \text{ inches}}{42 \text{ miles}} \qquad \text{Set up a proportion}$$

$$9 \cdot x = 1 \cdot 42 \qquad \text{Property of proportions}$$

$$9x = 42 \qquad \text{Multiply as indicated}$$

$$x = \frac{42}{9} \qquad \text{Divide each member by 9}$$

$$= \frac{14}{3} \text{ or } 4\frac{2}{3} \qquad \text{Reduce to lowest terms}$$

Thus, $4\frac{2}{3}$ inches represents 42 miles on the map.

67. A man earns $180 per week. How many weeks must he work to earn $1,260?

$\boxed{68.}$ An automobile uses 8 liters of gasoline to travel 84 kilometers. How many liters are needed to travel 1,428 kilometers?

69. If 24 grams of water will yield 4 grams of hydrogen, how many grams of hydrogen will there be in 216 grams of water?

70. The operating instructions for a gasoline chain saw call for a 16 gallons : 1 pint fuel-to-oil mixture. How many *pints* of oil are needed to mix with 88 gallons of fuel?

71. The power-to-weight ratio of a given engine is 5 : 3. What is the weight of the engine if it produces 650 horsepower?

72. If a 20-pound casting costs $1.50, at this same rate, how much would a 42-pound casting cost?

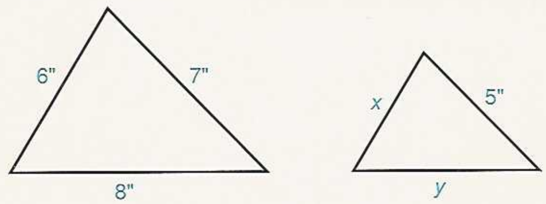
73. A copper wire 300 feet long has a resistance of 1,024 ohms. What is the resistance of 2,000 feet of copper wire?

74. In a hydraulic press, the force on the output piston is to the force on the input piston as the area of the output piston is to the area of the input piston. That is, $\frac{F_o}{F_i} = \frac{A_o}{A_i}$ or $F_o : F_i = A_o : A_i$. Find the area of the input piston if F_o is 15.2 pounds, F_i is 6.5 pounds, and A_o is 10.4 inches^2.

75. If the ratio of the wins to the losses of the Chicago Cubs in a given season is 6:5, how many games did they lose if they won 90 games?

$\boxed{76.}$ A rectangular picture that is 10 inches long and 8 inches wide is to be enlarged so that the enlargement will be 36 inches wide. What should be the length of the enlargement?

77. The corresponding sides of the triangles in the diagram are in proportion. Find the dimensions of the missing sides, x and y. (*Hint:* The corresponding sides are 6″ and x, 7″ and 5″, 8″ and y.)



78. Ann is operating a machine that can produce 14 parts in 20 minutes. How long will it take for her to produce 224 parts?

79. A punch machine can make 72 holes in 4 minutes. How many holes can the machine make in 3 hours?

80. A roof rises $6\frac{1}{2}$ feet in a rafter span of 9 feet. At this rate, what would be the rise in a 15-foot span?

81. Nat can type 3 pages of an English paper in 15 minutes. How long would it take him to type 54 pages? (State the answer in hours and minutes.)

$\boxed{82.}$ On a draftsman scale, $\frac{1}{8}$ inch represents 1 foot. What length will a measurement of $2\frac{5}{8}$ inches on the scale represent?

83. An automobile engine uses $\frac{3}{4}$ quart of oil in 900 miles. How much oil will it take in 3,000 miles?

Review exercises

Solve the following equations. See sections 2–4 and 2–7.

1. $6y + 5 = y - 4$ (Find the solution set.)
2. $P = 2\ell + 2w$ for w
3. The product of two consecutive odd integers is 143. Find the integers. See section 4–8.
4. Find the solution set of the inequality $3x - 2 \leq 2(x + 1)$. See section 2–7.

Completely factor each expression. See sections 4–2 and 4–4.

5. $16x^2 - y^2$
6. $x^2 - 16x - 17$
7. $5x^2 - 5x - 10$

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Chapter 5 lead-in problem

On a road map, 3 inches represent a distance of 45 miles. If the distance between Detroit and Sault Ste. Marie is 23 inches on the map, how far is it from Detroit to Sault Ste. Marie?

Solution

Let x = the distance from Detroit to Sault Ste. Marie. Then, since 3 inches represents 45 miles on the map, we use the relationship

3 in. is to 45 mi as 23 in. is to x mi

which we write as the proportion

$$\frac{3}{45} = \frac{23}{x}$$

$$3 \cdot x = 45 \cdot 23 \qquad \text{Property of proportions}$$

$$x = \frac{45 \cdot 23}{3} \qquad \text{Divide each member by 3}$$

$$x = 15 \cdot 23 \qquad \text{Reduce by 3}$$

$$x = 345 \qquad \text{Multiply in the right member}$$

The distance from Detroit to Sault Ste. Marie is 345 miles.

Chapter 5 summary

1. A **rational expression** can be written in the form $\frac{P}{Q}$, where P and Q are polynomials, $Q \neq 0$.
2. The **domain** of a rational expression in one variable is the set of all replacement values of the variable for which the rational expression is defined.
3. The **fundamental principle of rational expressions** is used to *reduce* rational expressions *to lowest terms* and to obtain equivalent rational expressions having the same denominator for addition and subtraction. It states, $\frac{PR}{QR} = \frac{P}{Q}$, where P, Q, and R are polynomials, Q and R are not equal to zero.
4. To **reduce** a rational expression to lowest terms, we divide the numerator and the denominator by any common factors.
5. To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.
6. To divide a polynomial by a polynomial, the dividend and the divisor must be arranged in descending powers of the same variable with zeros inserted for missing variables.
7. A **ratio** is the comparison of two numbers by division.
8. A **proportion** is a statement of equality of two ratios.
9. If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$, where ad and bc are called the cross products.

Chapter 5 error analysis

1. Finding the domain of a rational expression
   Example: $\frac{3x+1}{x^2+2x} = \frac{3x+1}{x(x+2)}$
   Domain is all real numbers except -2.
   Correct answer: Domain is all real numbers except 0 and -2.
   What error was made? (*see page 204*)

2. Reducing to lowest terms by "cancelling"
   Example: $\frac{x-3}{x^2-9} = \frac{\overset{1}{\cancel{x}} - \overset{1}{\cancel{3}}}{\underset{x}{\cancel{x^2}} - \underset{3}{\cancel{9}}} = \frac{1-1}{x-3} = \frac{0}{x-3} = 0$
   Correct answer: $\frac{1}{x+3}$
   What error was made? (*see page 208*)

3. Dividing a polynomial by a monomial
Example: $\frac{y - y^2}{y} = \frac{\overset{1}{\cancel{y}} - y^2}{\underset{1}{\cancel{y}}} = 1 - y^2$
Correct answer: $1 - y$
What error was made? (*see page 212*)
4. Dividing a polynomial by a polynomial
Example: $\frac{x^2 - 3x - 4}{x + 1}$

$$\begin{array}{r} x - 2 \\ x + 1 \overline{) x^2 - 3x - 4} \\ \underline{x^2 + x} \\ -2x - 4 \\ \underline{-2x - 2} \\ -6 \end{array} = x - 2 - \frac{6}{x + 1}$$

Correct answer: $x - 4$
What error was made? (*see page 213*)
5. Solving proportion problems
Example: On a map, 1 inch represents 10 miles. How many inches represent 24 miles? Let x = inches represented by 24 miles.
$\frac{1}{10} = \frac{24}{x}$
$x = 240$ inches
Correct answer: 2.4 inches
What error was made? (*see page 223*)
6. Order of operations
Example: $10 - 12 \div 2 + 6 = 5$
Correct answer: 10
What error was made? (*see page 57*)
7. Division using zero
Example: $\frac{0}{-3}$ is undefined
Correct answer: $\frac{0}{-3} = 0$
What error was made? (*see page 53*)
8. Combining like terms
Example: $8xy - 4xy = 4$
Correct answer: $4xy$
What error was made? (*see page 80*)
9. Reciprocal of a number
Example: The reciprocal of 1 is -1.
Correct answer: The reciprocal of 1 is itself.
What error was made? (*see page 94*)
10. Properties of exponents
Example: $(3x)^3 = 3x^3$
Correct answer: $27x^3$
What error was made? (*see page 129*)

Chapter 5 critical thinking

If m and n represent 2 integers where $n > m$, how many integers are there from m to n?

Chapter 5 review

[5–1]

Determine the domain of the given rational expression.

1. $\frac{x + 1}{x}$
2. $\frac{y - 3}{y + 7}$
3. $\frac{3x + 1}{x - 9}$
4. $\frac{2z - 5}{3z + 2}$
5. $\frac{x}{5x - 3}$
6. $\frac{x^2 - x + 4}{x^2 + x - 12}$
7. $\frac{x^2 + 3x + 2}{x^2 - 1}$

[5–2]

Reduce the following rational expressions to lowest terms. Assume no denominator is equal to zero.

8. $\frac{18ab^2}{6a^2b}$
9. $\frac{45x^2yz^3}{30xy^3z^2}$
10. $\frac{x^2 - 49}{x^2 + 14x + 49}$
11. $\frac{x^2 - 3x - 18}{x^2 + x - 42}$
12. $\frac{18a - 6b}{15a - 5b}$
13. $\frac{x^2 - y^2}{y - x}$
14. $\frac{3p^2 - 8p + 4}{5p^2 - 9p - 2}$
15. $\frac{2R^2 - 32}{6R^2 + 22R - 8}$
16. $\frac{20 - 9n + n^2}{8 + 2n - n^2}$

[5–3]

Find the indicated quotients.

17. $\dfrac{24x^3}{-3x}$

18. $\dfrac{2a^2 - 3a + 5a^3}{a}$

19. $\dfrac{5x^2y - 3xy^4 + x^2y^2}{xy}$

20. $\dfrac{8a^3b + 12a^2b^2 - 24a^3b^7}{4a^2b}$

21. $\dfrac{8a^2 - 2a - 3}{2a - 1}$

22. $\dfrac{3a^2 - 17a + 11}{a - 5}$

23. $\dfrac{x^2 - 49}{x + 7}$

24. $\dfrac{20x^3 - 19x^2 - 13x + 12}{4x - 3}$

[5–4]

Find the indicated ratios in two different forms reduced to lowest terms.

25. 15 meters to 35 meters

26. 36 pounds to 16 pounds

27. 12 inches to $2\frac{1}{2}$ feet

28. 450 miles to 15 gallons

29. In business, the current ratio compares current assets to current liabilities and represents the measure of the firm's ability to pay off the liabilities over a time period. What is the current ratio, reduced to lowest terms, if the firm's total current assets are \$4,386 and total current liabilities are \$1,762?

30. The May company has 42 sales representatives who are meeting their sales quota. If another 18 sales representatives have fallen short of their quota, what is the ratio of success to failure?

Find the value of the unknown that makes the statement a proportion.

31. $\dfrac{8}{x} = \dfrac{9}{36}$

32. $\dfrac{5.4}{3.6} = \dfrac{a}{2.4}$

33. $y : 18 = 15 : 25$

34. $\dfrac{5}{6} : \dfrac{1}{2} = \dfrac{2}{3} : p$

35. If a blueprint is drawn to the scale $\frac{1}{8}$ inch = 1 foot, what is the size of the corresponding part of a final product if the blueprint measurement is $4\frac{3}{8}$ inches?

36. An automobile has a 16-quart cooling system. If the ratio of antifreeze to water is 3 to 1, how much of each does the system have? (*Hint:* Let x be the amount of antifreeze. Then $16 - x$ is the amount of water.)

Chapter 5 cumulative test

Perform the indicated operations.

[1–8] **1.** $-4[7(12 - 2) - 8^3 + 3]$

[1–7] **2.** $\dfrac{(-6)(-8)}{(-2)(0)}$

[1–8] **3.** -6^2

Perform the indicated operations and simplify.

[2–3] **4.** $(5a - b) - [3a - (4b + 3a)]$

[3–2] **5.** $(3x - 2)^2$

[3–2] **6.** $(5y - 2)(5y + 2)$

[3–2] **7.** $(4a + 3b)(a - 6b)$

[3–2] **8.** $(x - y)^3$

[2–2] **9.** Given $a = -5$, $b = 3$, $c = 4$, and $d = -6$, evaluate the expression $(2a - 3b) - (5c + d)$.

Find the solution set.

[2–6] **10.** $3(x + 2) - 2(x - 4) = 12$

[2–6] **11.** $\dfrac{3x}{4} - 5 = 1$

[2–9] **12.** $-4 \le 2x + 5 < 11$

[2–9] **13.** $2(3x - 4) > 5(x - 1)$

Write in completely factored form.

[4–1] **14.** $x(m + n) - y(m + n)$

[4–3] **15.** $3a^2 + 7a + 4$

[4–4] **16.** $4x^2 - 20x + 25$

[4–4] **17.** $4z^2 - 9$

[4–4] **18.** $36 - y^2$

[4–2] **19.** $x^2 - 12x - 45$

[4–5] **20.** $2a^3 - 16b^3$

Find the solution set.

[4–7] **21.** $x^2 - 5x - 14 = 0$

[4–7] **22.** $2y^2 + 3y - 9 = 0$

Simplify and leave answers with only positive exponents.

[3–4] **23.** $(4yz^{-1})^{-3}$

[3–4] **24.** $\frac{a^{-7}}{a^{-10}}$

[3–4] **25.** $(4ab^2)(-2a^3b)(-a^{-1}b^{-3})$

[5–4] **26.** Find x when $36 : x = 21:14$.

[5–4] **27.** What is the ratio of 52 pounds to 24 pounds?

[5–3] **28.** Divide $(x^2 - 8x + 13) \div (x - 2)$.

Reduce the following expressions to lowest terms. Assume that no denominator is equal to zero.

[1–1] **29.** $\frac{56}{42}$

[3–3] **30.** $\frac{36ab^3}{28a^3b^2}$

[5–2] **31.** $\frac{a^2 - 36}{a^2 - a - 42}$

[5–2] **32.** $\frac{8x - 8y}{5x^2 - 5y^2}$

[5–2] **33.** $\frac{y^2 - y - 20}{y^2 - 25}$

[5–2] **34.** $\frac{2x^2 - 5xy - 3y^2}{6x^2 + 7xy + 2y^2}$

[5–4] **35.** A photograph that is 8 inches by 12 inches is to be enlarged. If the enlargement calls for the longest side to be 32 inches, how many inches should the other side be?

[5–4] **36.** Two lots are proportional in their lengths and widths. If the larger lot is 15 feet wide and 28 feet long and the length of the smaller lot is 24 feet, how wide is the smaller lot?

16. $(3R+1)(a+b)$ **17.** $(x-3y)(2a-3b)$ **18.** $(3a-b)(2x-y)$ **19.** $(x+2y)(4a+3b)$ **20.** $(a+3b)(x-4)$ **21.** $(x^2+4)(a-2b)$ **22.** $(x-7)(x-2)$ **23.** $2a(a-5)(a+1)$ **24.** $(a+12)(a+2)$ **25.** $(x-8)(x+4)$ **26.** $(a-18)(a+2)$ **27.** $3(x-5)(x+2)$ **28.** $x(x-3)(x+2)$ **29.** $x(x-7)(x+3)$ **30.** $(ab+3)(ab-2)$ **31.** $(ab+6)(ab+4)$ **32.** $(ab-6)(ab-3)$ **33.** $(ab-10)(ab+2)$ **34.** $(2x+1)(2x+1)$ or $(2x+1)^2$ **35.** $9(r-2)(r-2)$ or $9(r-2)^2$ **36.** $(4x-1)(x-1)$ **37.** $(3a+5)(3a-2)$ **38.** $(4a-3)(2a+1)$ **39.** $(6x+1)(4x+3)$ **40.** $(4a-3)(2a-3)$ **41.** $(2a+3)(a+6)$ **42.** $(2a+3)(2a-3)$ **43.** $(6b+c)(6b-c)$ **44.** $(5+a)(5-a)$ **45.** $4(2x+y)(2x-y)$ **46.** $(3x+y^2)(3x-y^2)$ **47.** $(x^2+4)(x+2)(x-2)$ **48.** $(y^2+9)(y+3)(y-3)$ **49.** $(b+6)^2$ **50.** $(c-5)^2$ **51.** $(2x-3)^2$ **52.** $(3x-2)^2$ **53.** $(R+2S)(R^2-2RS+4S^2)$ **54.** $2(2x-3)(4x^2+6x+9)$ **55.** $(3a+5b)(9a^2-15ab+25b^2)$ **56.** $(xy-1)(x^2y^2+xy+1)$ **57.** $2(x^3+5)(x^6-5x^3+25)$ **58.** $(4x^4-y^5)(16x^8+4x^4y^5+y^{10})$ **59.** $(ab^2+c^3)(a^2b^4-ab^2c^3+c^6)$ **60.** $3x^3(4x-1)$ **61.** $(a-5)(a+2)$ **62.** $(4a-1)(a-5)$ **63.** $(3y+2)(3y-2)$ **64.** $(2a+3b)(3x-2)$ **65.** $(b-5)(b+4)$ **66.** $(3x+2)(3x+5)$ **67.** $(a+7)^2$ **68.** $3x^3(2x+1)(2x-1)$ **69.** $c(c+4)(c+5)$ **70.** $(4a-1)^2$ **71.** $(b^2+1)(b+1)(b-1)$ **72.** $\{1,-3\}$ **73.** $\{0,8\}$ **74.** $\left\{-\frac{1}{5},\frac{7}{3}\right\}$ **75.** $\left\{\frac{1}{7},\frac{8}{5}\right\}$ **76.** $\left\{\frac{4}{3},9\right\}$ **77.** $\{0,-9,-4\}$ **78.** $\left\{\frac{4}{5},-\frac{4}{5}\right\}$ **79.** $\{-1,3,-2\}$ **80.** $\left\{0,\frac{9}{4}\right\}$ **81.** $\{-1,1\}$ **82.** $\{0,64\}$ **83.** $\{-5,5\}$ **84.** $\{6,-5\}$ **85.** $\{1\}$ **86.** $\left\{-\frac{1}{4},-3\right\}$ **87.** $\left\{-\frac{2}{5},2\right\}$ **88.** $\{4\}$ **89.** $\{1,3\}$ **90.** $\left\{-1,\frac{3}{4}\right\}$ **91.** $\{2,-3\}$ **92.** 9,10 **93.** 8 feet and 13 feet **94.** 20 cattle **95.** 6 seconds

Chapter 4 cumulative test

1. 33 **2.** $8a^6b^3$ **3.** $a^2+4ab+4b^2$ **4.** a^6 **5.** 12 **6.** x^5y^4 **7.** $5x-7y$ **8.** $9x^2-4y^2$ **9.** $\frac{8a^6}{b^3}$ **10.** $\frac{1}{x^2}$ **11.** $4x+4y$ **12.** $\frac{x^6}{9y^4}$ **13. a.** 0 **b.** 0 **c.** -6 **14.** $\left\{\frac{9}{5}\right\}$ **15.** $x<12$ **16.** $\{7\}$ **17.** $x\geq 3$ **18.** $\{-3,3\}$ **19.** $\{2,5\}$ **20.** $x<\frac{5}{2}$ **21.** $2\leq x\leq\frac{14}{3}$ **22.** $x=3y$ **23.** $x=\frac{5y+2}{3}$ **24.** $2ab(1-2ab-4a^2b^4)$ **25.** $(2a+3)^2$ **26.** $(5c+3d)(5c-3d)$ **27.** $(2a+3)(2a-5)$ **28.** $(x+3)(x+6)$ **29.** 14,39 **30.** 11,13 **31.** \$10,000 at 8%; \$5,000 at 6% **32.** 6 meters by 11 meters

Chapter 5

Exercise 5–1

Answers to odd-numbered problems

1. $\frac{1}{3}$ **3.** undefined **5.** 4 **7.** $\frac{13}{23}$ **9.** $\frac{16}{17}$ **11.** 3 **13.** undefined **15.** $\frac{15}{2}$ **17.** all real numbers except 0 **19.** all real numbers except 5 **21.** all real numbers except -3 **23.** all real numbers except $\frac{3}{4}$ **25.** all real numbers except $\frac{8}{3}$ **27.** all real numbers except 1 and 6 **29.** all real numbers except -2 and $\frac{4}{3}$ **31.** all real numbers except $-\frac{2}{3}$ and $\frac{2}{3}$ **33.** all real numbers except -3 and 3 **35.** all real numbers except $\frac{7}{3}$ **37.** all real numbers except -1 and 1 **39.** all real numbers **41.** $L=0$ **43.** $T_1\neq 0$, $T_2\neq 0$

Solutions to trial exercise problems

6. $\frac{-5b^3}{5-2b}$; $b=-2$

$$\frac{-5b^3}{5-2b}=\frac{-5(-2)^3}{5-2(-2)}=\frac{-5(-8)}{5+4}=\frac{40}{9}$$

9. $\frac{(-2x)^2}{x^2+3x+7}$; $x=2$

$$\frac{(-2x)^2}{x^2+3x+7}=\frac{[(-2)(2)]^2}{(2)^2+3(2)+7}=\frac{(-4)^2}{4+6+7}=\frac{16}{17}$$

22. $\frac{x+1}{2x-1}$ Set $2x-1=0$, then $2x=1$ and $x=\frac{1}{2}$.

Domain is all real numbers except $\frac{1}{2}$.

25. $\frac{y+4}{8-3y}$ Set $8-3y=0$, then $3y=8$ and $y=\frac{8}{3}$.

Domain is all real numbers except $\frac{8}{3}$.

28. $\frac{5s^2+7}{2s^2-s-3}$ Set $2s^2-s-3=0$ and factor. We have

$(2s-3)(s+1)=0$. Then

$$\begin{aligned} 2s-3&=0 &\quad\text{or } s+1&=0\\ 2s&=3 & s&=-1\\ s&=\frac{3}{2} & s&=-1 \end{aligned}$$

Domain is all real numbers except -1 and $\frac{3}{2}$.

32. $\frac{a-2}{4a^2-16}$ Set $4a^2-16=0$ and factor. We have

$4(a-2)(a+2)=0$.

This is true if and only if $a-2=0$, $a=2$ or $a+2=0$, $a=-2$.

Domain is all real numbers except -2 and 2.

37. $\frac{17q}{3q^2-3}$ Set $3q^2-3=0$ and factor to get

$3(q^2-1)=0$

$3(q+1)(q-1)=0$

$q+1=0$ or $q-1=0$

Then $q=-1$ or $q=1$

Domain is all real numbers except -1 and 1.

Review exercises

1. distributive property **2.** -50 **3.** 5 **4.** $\frac{5}{9}$

5. $(2x + 1)(x - 5)$ **6.** $4(y - 5)^2$

Exercise 5–2

Answers to odd-numbered problems

1. $\frac{3}{4}$ **3.** $\frac{2x}{5}$ **5.** $\frac{4x}{3}$ **7.** $-\frac{4}{3x^2}$ **9.** $\frac{4a}{5b}$ **11.** -5 **13.** $\frac{5}{4}$

15. $\frac{6}{x+3}$ **17.** $\frac{1}{a-b}$ **19.** $\frac{3}{5}$ **21.** $\frac{x-1}{2(x+1)}$ **23.** $\frac{x-3}{x+3}$

25. $\frac{x-5}{x-3}$ **27.** $\frac{2y+3}{4y-1}$ **29.** $\frac{1}{x^2+3x+9}$ **31.** $\frac{a-b}{a^2-ab+b^2}$

33. -4 **35.** $-\frac{2}{3}$ **37.** $-2(x+y)$ **39.** $\frac{y-x}{x+y}$ **41.** $\frac{n-m}{n+m}$

43. $-\frac{4}{x+y}$ **45.** $-\frac{1}{x+4}$

Solutions to trial exercise problems

10. $\frac{15a^2x^3}{35ax^2} = \frac{3 \cdot 5 \cdot a \cdot a \cdot x \cdot x \cdot x}{7 \cdot 5 \cdot a \cdot x \cdot x} = \frac{3 \cdot a \cdot x \cdot (5 \cdot a \cdot x \cdot x)}{7 \cdot (5 \cdot a \cdot x \cdot x)} = \frac{3ax}{7}$

17. $\frac{a+b}{a^2-b^2} = \frac{a+b}{(a+b)(a-b)} = \frac{1}{a-b}$

25. $\frac{x^2-3x-10}{x^2-x-6} = \frac{(x-5)(x+2)}{(x-3)(x+2)} = \frac{x-5}{x-3}$

30. $\frac{x+2}{x^3+8} = \frac{x+2}{(x+2)(x^2-2x+4)} = \frac{1}{x^2-2x+4}$

34. $\frac{8b-8a}{a-b} = \frac{-8(a-b)}{a-b} = -8$

45. $\frac{x-3}{12-x-x^2} = \frac{x-3}{(3-x)(4+x)} = \frac{-1(3-x)}{(3-x)(4+x)} = \frac{-1}{x+4}$

Review exercises

1. 3.14×10^{-4} **2.** $-\frac{11}{7}$ **3.** 5 ft and 11 ft **4.** 1 **5.** $27y^3$

6. $-8x^9y^6$

Exercise 5–3

Answers to odd-numbered problems

1. $4x^2$ **3.** $-5x^3yz$ **5.** $3(a-b)$ **7.** $2a(b-c)$

9. $-4a^2b(x+y)$ **11.** $-4a^2+2a$ **13.** a^2-3a+2

15. $5a^2-3a+4-\frac{2}{a}$ **17.** $-x+y+2y^2$

19. $10xy^2+7-6y^2$ **21.** $a-c$ **23.** $a+9+\frac{28}{a-2}$

25. $a+2+\frac{4}{a+3}$ **27.** $a+3+\frac{1}{a+3}$

29. $3a-4-\frac{4}{3a-4}$ **31.** x^2+2x+4 **33.** x^2+2x+3

35. $b^2+7b+14+\frac{6}{b-1}$ **37.** $3a+8$

39. $x^2-1+\frac{6x-13}{x^2+3x-5}$ **41.** $y^2-2y+9+\frac{-26y+42}{y^2+2y-5}$

43. $4x+3$ **45.** $-6x^4+15x^3+4x^2-22x+30$

Solutions to trial exercise problems

5. $\frac{3(a-b)^2}{a-b} = \frac{3(a-b)^2}{(a-b)} = 3(a-b)^{2-1} = 3(a-b)^1 = 3(a-b)$ or $3a-3b$ (*Note:* A quantity is treated as just one term.)

15. $\frac{15a^3-9a^2+12a-6}{3a} = \frac{15a^3}{3a} - \frac{9a^2}{3a} + \frac{12a}{3a} - \frac{6}{3a}$

$= 5a^2-3a+4-\frac{2}{a}$

17. $\frac{x^2y-xy^2-2xy^3}{-xy} = \frac{x^2y}{-xy} - \frac{xy^2}{-xy} - \frac{2xy^3}{-xy} = -x+y+2y^2$

21. $\frac{a(b-1)-c(b-1)}{b-1} = \frac{a(b-1)}{(b-1)} - \frac{c(b-1)}{(b-1)} = a-c$

30. $(27a^3-1) \div (3a-1)$

(*Note:* Insert zeros to hold positions where terms are missing.)

$$
\begin{array}{r}
9a^2+3a+1 \\
3a-1\,\overline{)\,27a^3+0a^2+0a-1} \\
\underline{27a^3-9a^2} \\
9a^2+0a \\
\underline{9a^2-3a} \\
3a-1 \\
\underline{3a-1} \\
0
\end{array}
$$

Answer: $9a^2+3a+1$

42. The length times the width is x^2+6x+8. If we know the length to be $x+4$, then the width is found by

$$
\begin{array}{r}
x+2 \\
x+4\,\overline{)\,x^2+6x+8} \\
\underline{x^2+4x} \\
2x+8 \\
\underline{2x+8} \\
0
\end{array}
$$

Width $= x+2$

Review exercises

1. $\left\{-2, -\frac{1}{4}\right\}$ **2.** 6 ft and 7 ft **3.** $16x^2-24x+9$

4. x^3+3x^2+x-2 **5.** $25y^2-1$ **6.** $\frac{3y+1}{2y+3}$

Exercise 5–4

Answers to odd-numbered problems

1. $\frac{12}{7}$; 12:7 **3.** $\frac{1}{6}$; 1:6 **5.** $\frac{8}{3}$; 8:3 **7.** $\frac{3}{1}$; 3:1

9. $\frac{32}{3}$; 32:3 **11.** $\frac{1}{4}$; 1:4 **13.** $\frac{5}{4}$; 5:4 **15.** $\frac{39}{32}$; 39:32

17. $\frac{2}{1}$; 2:1 **19.** $\frac{3}{7}$; 3:7 **21.** $\frac{3}{7}$; 3:7 **23.** $\frac{5}{2}$; 5:2

25. $\frac{3}{2}$; 3:2 **27.** $\frac{5}{12}$; 5:12 **29.** $\frac{60}{7}$; 60:7 **31.** $\frac{1}{26}$; 1:26

33. $\frac{2 \text{ lb}}{1 \text{ ft}^3} = 2 \text{ lb:1 ft}^3$ **35.** $\frac{8}{1}\left(\frac{\text{g}}{\text{cm}^3}\right)$; 8 g:1 cm^3

37. $\frac{60}{1}\left(\frac{\text{miles}}{\text{hr}}\right)$; 60 miles:1 hr **39.** $\frac{15}{17}$ **41.** $\frac{5}{12}; \frac{12}{5}$

43. $\frac{8}{3}$ **45.** $\frac{13}{14}$ **47.** $\frac{21}{2}$ **49.** $\frac{31}{2}$ **51.** $\frac{7}{50}$

53. 7 ft-lb/sec **55.** $x = \frac{5}{4}$ **57.** $p = \frac{100}{9}$ **59.** $x = \frac{16}{5}$

61. $a = \frac{12}{19}$ **63.** $x = 3$ **65.** $a = \frac{50}{13}$ **67.** 7 weeks

69. 36 grams of hydrogen **71.** 390 **73.** $6{,}826\frac{2}{3}$ ohms

75. 75 losses **77.** $x = \frac{30}{7}$ inches, $y = \frac{40}{7}$ inches

79. 3,240 holes **81.** 270 minutes $= 4\frac{1}{2}$ hours **83.** $2\frac{1}{2}$ quarts

Solutions to trial exercise problems

9. 8 to $\frac{3}{4}$ is written $\dfrac{8}{\frac{3}{4}}$ or $8{:}\frac{3}{4}$

but $\dfrac{8}{\frac{3}{4}} = 8 \cdot \frac{4}{3} = \frac{32}{3}$

Therefore we have the ratio $\frac{32}{3}$ or 32:3

13. $\frac{5}{6}$ to $\frac{2}{3}$ is written $\dfrac{\frac{5}{6}}{\frac{2}{3}} = \frac{5}{6} \cdot \frac{3}{2} = \frac{5}{4}$

Therefore we have the ratio $\frac{5}{4}$ or 5:4

17. 4.2 to 2.1 is written $\frac{4.2}{2.1} = \frac{42}{21} = \frac{2}{1}$

so we have the ratio $\frac{2}{1}$ or 2:1

27. Since there are $3 \cdot 12 = 36$ inches in 3 feet, we have

15 inches to 36 inches written $\frac{15}{36} = \frac{5}{12}$ or 5:12

36. 300 miles to 10 gallons is written $\dfrac{300 \text{ miles}}{10 \text{ gallons}} = \dfrac{30 \text{ miles}}{1 \text{ gallon}}$,

which we state as 30 miles per gallon.

39. $ME = \dfrac{\text{output}}{\text{input}} = \dfrac{375}{425} = \dfrac{15}{17}$

Therefore the mechanical efficiency is $\frac{15}{17}$

44. $\dfrac{\text{Cutting speed of tool steel}}{\text{Cutting speed of cast iron}} = \dfrac{20 \text{ ft/min}}{45 \text{ ft/min}} = \dfrac{4}{9}$

53. $\dfrac{F}{t} = \dfrac{42 \text{ ft-lb}}{6 \text{ sec}} = 7$ ft-lb per sec

55. $\frac{9}{x} = \frac{36}{5}$, then $36 \cdot x = 9 \cdot 5$

$$36x = 45$$

$$x = \frac{45}{36} = \frac{5}{4}$$

Therefore $\dfrac{9}{\frac{5}{4}} = \dfrac{36}{5}$

59. $6{:}15 = x{:}8$, then $15 \cdot x = 6 \cdot 8$

$$15x = 48$$

$$x = \frac{48}{15} = \frac{16}{5}$$

Therefore $6{:}15 = \frac{16}{5}{:}\,8$

62. $\frac{3}{4} : 4 = \frac{1}{2} : b$, then $\frac{3}{4} \cdot b = 4 \cdot \frac{1}{2}$

$$\frac{3}{4}b = 2$$

$$b = 2 \cdot \frac{4}{3} = \frac{8}{3}$$

so $\frac{3}{4} : 4 = \frac{1}{2} : \frac{8}{3}$

68. Let x = the number of liters of gasoline to travel 1,428 kilometers.

Then $\dfrac{8 \text{ liters}}{84 \text{ km}} = \dfrac{x \text{ liters}}{1{,}428 \text{ km}}$ and

$$x \cdot 84 = 8 \cdot 1{,}428$$

$$84x = 11{,}424$$

$$x = \frac{11{,}424}{84} = 136$$

Therefore at the same rate of gasoline consumption, it would take 136 liters to travel 1,428 kilometers.

76. Let ℓ = the length of the enlargement.

Then $\dfrac{10 \text{ in.}}{8 \text{ in.}} = \dfrac{\ell \text{ in.}}{36 \text{ in.}}$ and $8 \cdot \ell = 10 \cdot 36$

$$8\ell = 360$$

$$\ell = \frac{360}{8} = 45$$

Therefore the enlargement will be 45 inches long.

82. Let x = the number of feet represented by $2\frac{5}{8}$ inches.

Then $\dfrac{\frac{1}{8} \text{ in.}}{1 \text{ ft}} = \dfrac{2\frac{5}{8} \text{ in.}}{x \text{ ft}}$ and $\frac{1}{8} \cdot x = 1 \cdot 2\frac{5}{8}$

$$\frac{1}{8}x = 2\frac{5}{8}$$

$$x = \frac{21}{8} \cdot 8 = 21$$

Therefore $2\frac{5}{8}$ inches represents 21 feet.

Review exercises

1. $\left\{-\frac{9}{5}\right\}$ **2.** $w = \dfrac{P - 2\ell}{2}$ **3.** 11 and 13; -13 and -11
4. $x \le 4$ **5.** $(4x + y)(4x - y)$ **6.** $(x - 17)(x + 1)$
7. $5(x - 2)(x + 1)$

Chapter 5 review

1. All real numbers except 0 **2.** All real numbers except -7
3. All real numbers except 9 **4.** All real numbers except $-\frac{2}{3}$
5. All real numbers except $\frac{3}{5}$ **6.** All real numbers except -4 and 3 **7.** All real numbers except -1 and 1 **8.** $\dfrac{3b}{a}$ **9.** $\dfrac{3xz}{2y^2}$
10. $\dfrac{x - 7}{x + 7}$ **11.** $\dfrac{x + 3}{x + 7}$ **12.** $\dfrac{6}{5}$ **13.** $-(x + y)$ **14.** $\dfrac{3p - 2}{5p + 1}$

15. $\dfrac{R-4}{3R-1}$ **16.** $\dfrac{5-n}{2+n}$ **17.** $-8x^2$ **18.** $2a-3+5a^2$
19. $5x-3y^3+xy$ **20.** $2a+3b-6ab^6$
21. $4a+1+\dfrac{-2}{2a-1}$ **22.** $3a-2+\dfrac{1}{a-5}$ **23.** $x-7$
24. $5x^2-x-4$ **25.** $\dfrac{3}{7}$ or 3:7 **26.** $\dfrac{9}{4}$ or 9:4 **27.** $\dfrac{2}{5}$ or 2:5
28. $30\dfrac{\text{miles}}{\text{gallon}}$ or 30 miles per gallon **29.** $\dfrac{2{,}193}{881}$ or 2,193:881
30. $\dfrac{7}{3}$ or 7:3 **31.** $x=32$ **32.** $a=3.6$ **33.** $y=\dfrac{54}{5}$
34. $p=\dfrac{2}{5}$ **35.** 35 feet **36.** 12 quarts antifreeze, 4 quarts water

Chapter 5 cumulative test

1. 1,756 **2.** undefined **3.** -36 **4.** $5a+3b$
5. $9x^2-12x+4$ **6.** $25y^2-4$ **7.** $4a^2-21ab-18b^2$
8. $x^3-3x^2y+3xy^2-y^3$ **9.** -33 **10.** $\{-2\}$ **11.** $\{8\}$
12. $-\dfrac{9}{2}\le x<3$ **13.** $x>3$ **14.** $(m+n)(x-y)$
15. $(3a+4)(a+1)$ **16.** $(2x-5)^2$ **17.** $(2z+3)(2z-3)$
18. $(6-y)(6+y)$ **19.** $(x-15)(x+3)$
20. $2(a-2b)(a^2+2ab+4b^2)$ **21.** $\{7,-2\}$ **22.** $\left\{\dfrac{3}{2},-3\right\}$
23. $\dfrac{z^3}{64y^3}$ **24.** a^3 **25.** $8a^3$ **26.** $x=24$ **27.** $\dfrac{13}{6}$ or 13:6
28. $x-6+\dfrac{1}{x-2}$ **29.** $\dfrac{4}{3}$ **30.** $\dfrac{9b}{7a^2}$ **31.** $\dfrac{a-6}{a-7}$
32. $\dfrac{8}{5(x+y)}$ **33.** $\dfrac{y+4}{y+5}$ **34.** $\dfrac{x-3y}{3x+2y}$ **35.** $21\frac{1}{3}$ inches
36. $12\frac{6}{7}$ feet

Chapter 6

Exercise 6–1

Answers to odd-numbered problems

1. $\dfrac{3}{5}$ **3.** $\dfrac{5}{6}$ **5.** $2a$ **7.** 10 **9.** $\dfrac{x}{4y}$ **11.** $\dfrac{3x}{4}$ **13.** $\dfrac{4}{35x}$
15. $\dfrac{35x}{4}$ **17.** $\dfrac{7c}{2b}$ **19.** $\dfrac{x}{6y^2}$ **21.** $\dfrac{16bcx}{3az}$ **23.** $\dfrac{2a}{5b^2x}$
25. $\dfrac{4}{x+y}$ **27.** $-\dfrac{3}{4}$ **29.** $-\dfrac{15}{4}$ **31.** $\dfrac{4}{a-5}$ **33.** $\dfrac{18(x-2)}{x+2}$
35. $\dfrac{24y(x-2)}{25}$ **37.** $(r-4)(r-1)$ **39.** $-4(x+3)$
41. $\dfrac{a-3}{a-5}$ **43.** $\dfrac{(x-3)(x+1)}{(x-1)(x+2)}$ **45.** $\dfrac{(2x-1)(x-1)}{(x-8)(x+7)}$
47. 1 **49.** $3x+4$ **51.** $\dfrac{1}{(2x+1)^2}$ **53.** $\dfrac{2a+12}{3a^2+9a+27}$
55. $\dfrac{(z-7)(z+3)}{5(z^2-2z+4)}$ **57.** $y+5$

Solutions to trial exercise problems

10. $\dfrac{7a}{12b}\cdot\dfrac{9b}{28}=\dfrac{7a\cdot 9b}{12b\cdot 28}=\dfrac{7\cdot a\cdot 3\cdot 3\cdot b}{2\cdot 2\cdot 3\cdot b\cdot 2\cdot 2\cdot 7}$
$=\dfrac{3\cdot a\cdot(7\cdot 3\cdot b)}{2\cdot 2\cdot 2\cdot 2\cdot(7\cdot 3\cdot b)}=\dfrac{3a}{16}$

20. $\dfrac{28m}{15n}\div\dfrac{7m^2}{3n^3}=\dfrac{28m\cdot 3n^3}{15n\cdot 7m^2}=\dfrac{4n^2}{5m}$

21. $\dfrac{24abc}{7xyz^2}\cdot\dfrac{14x^2yz}{9a^2}=\dfrac{3\cdot 8\cdot a\cdot b\cdot c\cdot 2\cdot 7\cdot x^2\cdot y\cdot z}{7\cdot x\cdot y\cdot z^2\cdot 3\cdot 3\cdot a^2}$
$=\dfrac{8\cdot b\cdot c\cdot 2\cdot x(3\cdot 7\cdot a\cdot x\cdot y\cdot z)}{z\cdot 3\cdot a(3\cdot 7\cdot a\cdot x\cdot y\cdot z)}=\dfrac{8\cdot b\cdot c\cdot 2\cdot x}{z\cdot 3\cdot a}=\dfrac{16bcx}{3az}$

25. $\dfrac{x+y}{3}\cdot\dfrac{12}{(x+y)^2}=\dfrac{2\cdot 2\cdot 3(x+y)}{3(x+y)^2}=\dfrac{4}{x+y}$

30. $\dfrac{8y+16}{3-y}\cdot\dfrac{4y-12}{3y+6}=\dfrac{8(y+2)\cdot 4(y-3)}{-(y-3)\cdot 3(y+2)}=\dfrac{8\cdot 4}{-3}=-\dfrac{32}{3}$

37. $\dfrac{r^2-16}{r+1}\div\dfrac{r+4}{r^2-1}=\dfrac{(r^2-16)(r^2-1)}{(r+1)(r+4)}$
$=\dfrac{(r-4)(r+4)(r+1)(r-1)}{(r+1)(r+4)}=\dfrac{(r-4)(r-1)}{1}$
$=r^2-5r+4$

39. $\dfrac{9-x^2}{x+y}\cdot\dfrac{4x+4y}{x-3}=\dfrac{(3-x)(3+x)\cdot 4(x+y)}{(x+y)(x-3)}$
$=\dfrac{-1(x-3)(x+3)\cdot 4(x+y)}{(x+y)(x-3)}=\dfrac{-1(x+3)\cdot 4}{1}$
$=-4(x+3)=-4x-12$

41. $\dfrac{a^2-5a+6}{a^2-9a+20}\cdot\dfrac{a^2-5a+4}{a^2-3a+2}$
$=\dfrac{(a-3)(a-2)\cdot(a-4)(a-1)}{(a-4)(a-5)\cdot(a-2)(a-1)}=\dfrac{a-3}{a-5}$

47. $\dfrac{6r^2-r-7}{12r^2+16r-35}\div\dfrac{r^2-r-2}{2r^2+r-10}$
$=\dfrac{(6r^2-r-7)(2r^2+r-10)}{(12r^2+16r-35)(r^2-r-2)}$
$=\dfrac{(6r-7)(r+1)(2r+5)(r-2)}{(6r-7)(2r+5)(r-2)(r+1)}=1$

49. $(3x^2-2x-8)\div\dfrac{x^2-4}{x+2}=\dfrac{(3x+4)(x-2)}{1}$
$\cdot\dfrac{x+2}{(x+2)(x-2)}=3x+4$

53. $\dfrac{10}{a^3-27}\cdot\dfrac{a^2+3a-18}{15}=\dfrac{2\cdot 5(a+6)(a-3)}{3\cdot 5(a-3)(a^2+3a+9)}$
$=\dfrac{2(a+6)}{3(a^2+3a+9)}=\dfrac{2a+12}{3a^2+9a+27}$

Review exercises

1. $\dfrac{19}{12}$ **2.** $\dfrac{11}{24}$ **3.** $2(x+5)(x-5)$ **4.** $(x+11)(x-2)$
5. $(x+4)^2$ **6.** $x=\dfrac{24}{5}$ **7.** $y=15$ **8.** 7.89×10^{-5}

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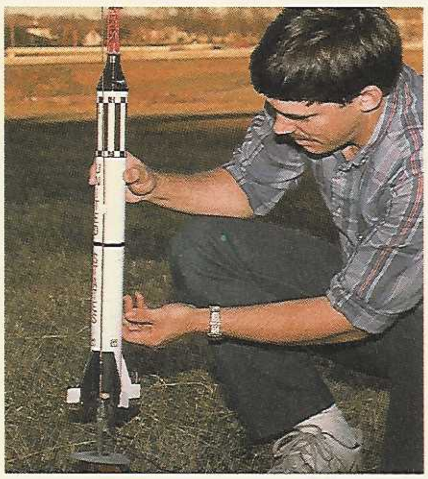

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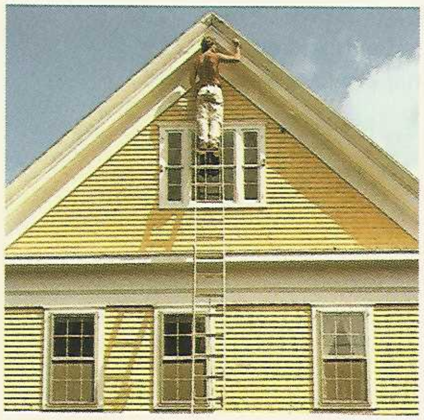

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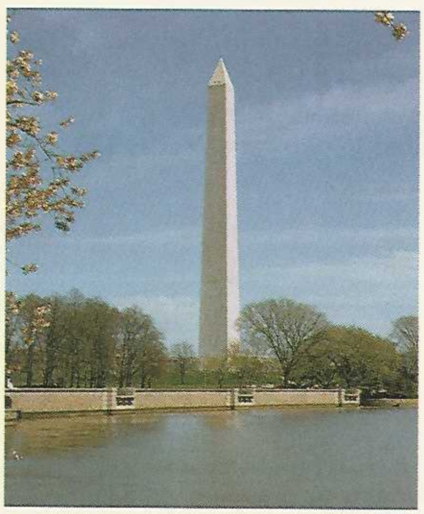

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